Nice spaces with bad reputations and political correctness of the mathematical language

Oleg Viro

May 20, 2011

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it distorts the ways we do mathematics.

Differential Spaces

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Inspired by H.Weyl's book on Riemann surfaces Die Idee der Riemannschen Fläche published in 1913.

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3. Juan A. Navarro Gonzales, Juan B. Sanch de Salas, $\mathcal{C}^\infty$ *-Differential Spaces*, Springer, 2003.

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In other words, $f \in \mathcal{C}^r(X)$ if for each $a \in X$ there exist $g, h \in \mathcal{C}^r(X)$ such that $h(a) > 0$ and $f(x) = g(x)$ for each x with $h(x) > 0$.

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4. *Topological space.* A topological space X with all continuous functions $X \to \mathbb{R}$.

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Isomorphisms of this category are called C^r*-diffeomorphims*.

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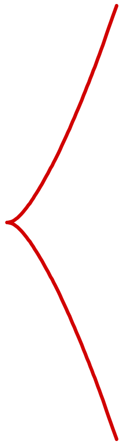

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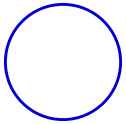
If so, how does the the image look like? Like this:  ?

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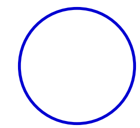
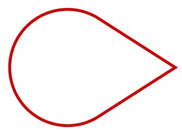
If so, how does the the image look like? Like this:  ?

No, like this:  !

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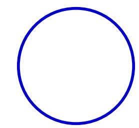
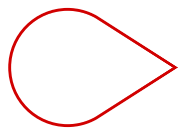
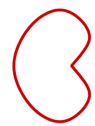
If so, how does the the image look like? Like this:  ?

No, like this:  !   Or that:  !

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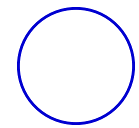
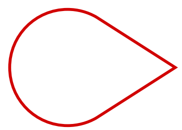
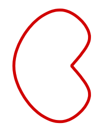
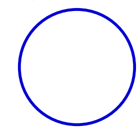
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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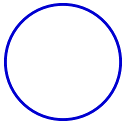
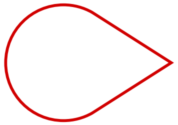
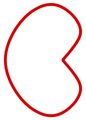
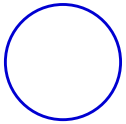
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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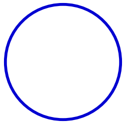
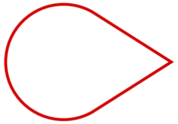
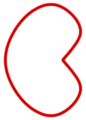
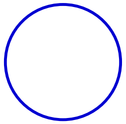
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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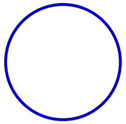
Then we get really a space diffeomorphic to  .

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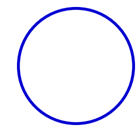
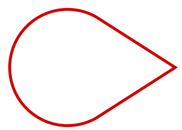
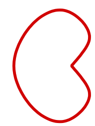
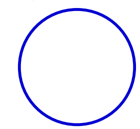
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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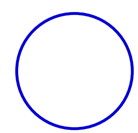
Then we get really a space diffeomorphic to  .

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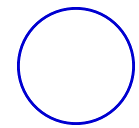
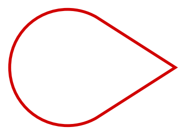
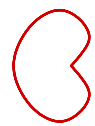
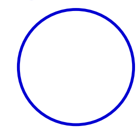
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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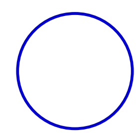
Then we get really a space diffeomorphic to  .

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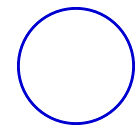
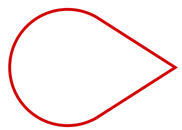
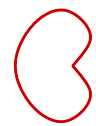
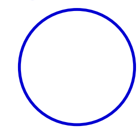
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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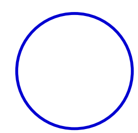
Then we get really a space diffeomorphic to  .

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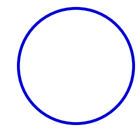
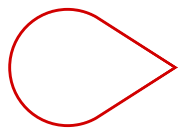
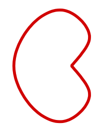
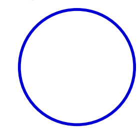
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

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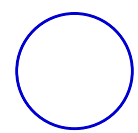
Then we get really a space diffeomorphic to  .

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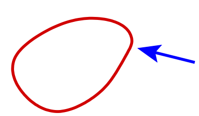
The resulting space:  .

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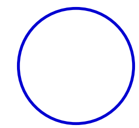
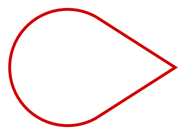
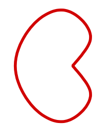
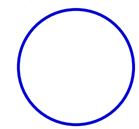
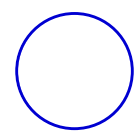
If so, how does the the image look like? Like this:  ?

No, like this:  ! Or that:  ! But not this:  !

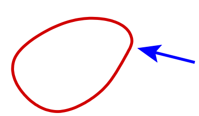
2. What if we take $[0,1.5]$ and identify each $x \in [0,0.5]$ with $x+1$?

Then we get really a space diffeomorphic to  .

New factorization:

Identifying end points of $[0,1]$, identify also tangent vectors!

That is consider functions $f:[0,1] \to \mathbb{R}$ with $f(0)=f(1)$ and $f'(0)=f'(1)$.

The resulting space:  . Smooth, but with jump of curvature.

Tangent vectors and dimensions

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Metric spaces

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Let X be a metric space. A function $f : X \to \mathbb{R}$ is *differentiable* at $p \in X$ if for any neighborhood U of p there exist points $q_1, \dots, q_n \in U$ and real numbers a_1, ..., a_n such that

$$\frac{|f(x) - f(p) - \sum a_i(\mathrm{dist}(q_i, x) - \mathrm{dist}(q_i, p))|}{\mathrm{dist}(x, p)} \to 0$$

as $x \to p$.

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Is this definition good?
At least, it recovers the smooth structure of a Riemannian manifold.

Finite Topological Spaces

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At early days of topology, they were the main objects of the
Combinatorial Topology

.

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Especially if the partition was a triangulation.

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the intersection of all of its neighborhoods.

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The star $St(\sigma)$ of a face σ is the union of all faces Σ
such that $\partial\Sigma \supset \sigma$.

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This partial order defines and is defined by the topology of Q.

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Corollary. *Any compact polyhedron is weak homotopy equivalent to a finite topological space.*

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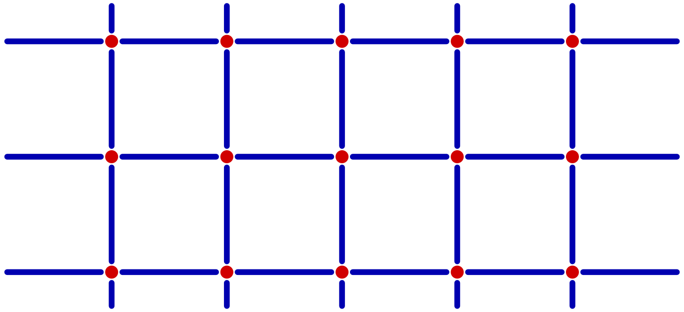

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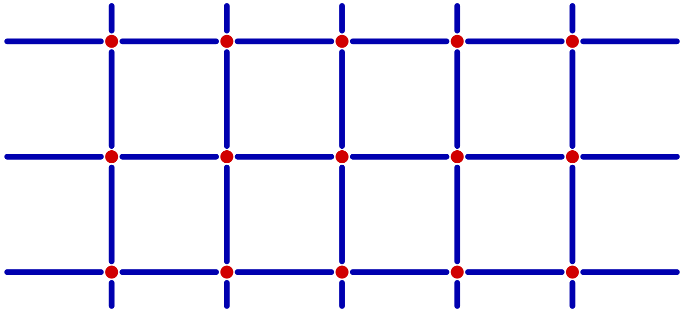

Digital circle of length d is the quotient space of the circle $S^1 \subset \mathbb{C}$ by the partition formed by complex roots of unity of degree d and open arcs connecting the roots next to each other.

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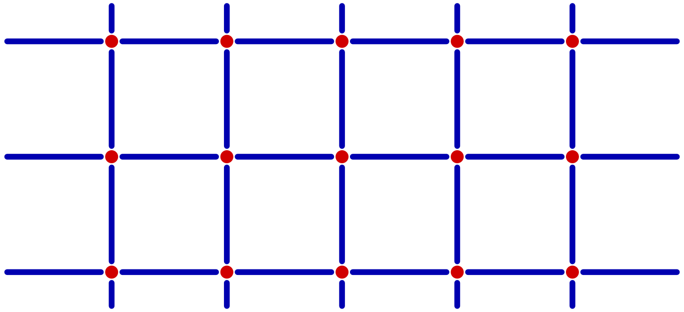
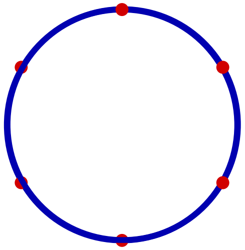


Digital circle



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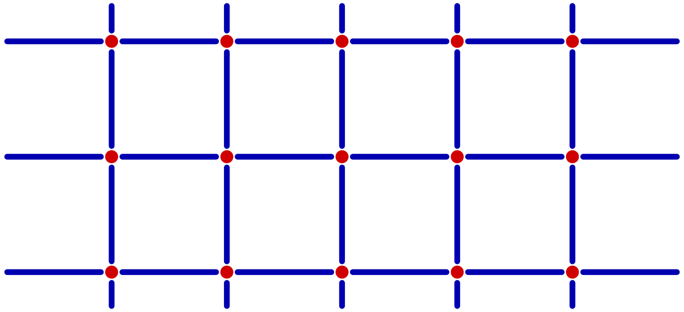

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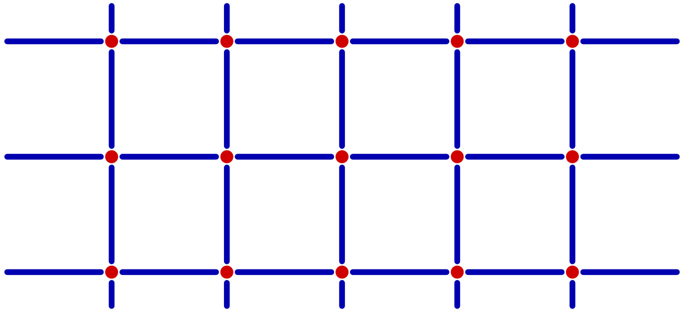

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Here spaces are not finite, but *locally finite*.

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(if x and y are T_0-equivalent, then both $x \in \operatorname{Cl} y$ and $y \in \operatorname{Cl} x$).

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Theorem. *Any finite topological space is weak homotopy equivalent to a compact polyhedron.*

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