Stony Brook University **Topology, Geometry I**
Mathematics Department MAT 530
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Homework 4

1. Prove that if a topological space X contains compact sets A and B such that $A \cap B$ is not compact, then X is not Hausdorff.

2. Find a topological space X and compact sets $A, B \subset X$ such that $A \cap B$ is not compact.

3. Prove that the graph of any continuous function $[0, 1] \to \mathbb{R}$ is closed, connected, path-connected, Hausdorff and compact.

4. Let X be a topological space consisting of 3 points, a, b and c. Let points a and c be closed and point b everywhere dense. Is X path-connected?

5. Describe a topology on the set of all natural numbers $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ such that with this topology $\mathbb{N}$ is Hausdorff and compact, and subspace $\mathbb{N} \smallsetminus \{1\}$ is a discrete. Is such topology unique?