Homework 2

1. Revisit all the definitions in sections 2, 3, 4, 5, 6, 10 and 11. Be prepared for reproducing any of the definitions in writing.

2. Prove that a map $f : X \to Y$ is continuous if and only if $\operatorname{Cl} f(A) \supset f(\operatorname{Cl} A)$ for any set $A \subset X$.

3. In an arbitrary metric space, what is relation between the boundary of an open ball $B_r(c)$ and sphere $S_r(c)$ of the same radius and center? Do they coincide? Or one of them is contained in the other? Give complete proofs for your answer.

4. Complete the solution of the Kuratowki problem started in the last lecture. Prove that $\operatorname{Cl} \operatorname{Int} \operatorname{Cl} \operatorname{Int} A = \operatorname{Cl} \operatorname{Int} A$ and $\operatorname{Int} \operatorname{Cl} \operatorname{Int} \operatorname{Cl} A = \operatorname{Int} \operatorname{Cl} A$.

5. Investigate relations between

- $\operatorname{Cl} A \cap \operatorname{Cl} B$ and $\operatorname{Cl}(A \cap B)$,
- $\operatorname{Cl} A \cup \operatorname{Cl} B$ and $\operatorname{Cl}(A \cup B)$.