Stony Brook University
Mathematics Department
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Geometry, Topology I
MAT 530
October 10, 2013

Program for the midterm exam

Topological structure in a set. Axioms of topological structure. Open and closed sets. Neighborhoods. Bases of a topological structure. Tests for a collection of subsets for being a base of a topology.

Continuous maps - definition and main properties. Local continuity and its relation to continuity. Fundamental covers. Continuity of a map continuous on elements of a fundamental cover.

Topological structures on a set induced by maps to or from topological spaces. Subspaces of a topological space. Relativity of openness. Quotient space. Continuity of a quotient map.

Homeomorphisms - definition and main properties. Homeomorphic spaces. Homeomorphism problem and topological properties. Topological embeddings.

Metric spaces. Axioms of metric. Balls and spheres. Topological structure defined by a metric.

Interior, boundary and exterior points. Interior, exterior, closure and boundary of a set. Everywhere dense set. Test for everywhere dense set.

Connectedness - definitions and their equivalence. Connected sets and their properties. Connected components. Connectedness and continuous maps. Connectedness of real line. Intermediate value theorem and its generalizations. Applications of connectedness to the homeomorphism problem.

Paths. Path connected spaces. Path connected sets. Path connected components. Path connectedness and continuous maps. Relation between connectedness and path connectedness.

Hausdorff axiom and uniqueness of limit. The first separation axiom and closedness of finite sets. Third and fourth separation axioms. Relations between separation axioms. Proofs of separation axioms for metric spaces.

Countability axioms. Second countability and separability. Lindelöf theorem. Second countability of a metric separable space. Bases at a point and first countability. Sequential approach to topology and continuity.

Compactness definitions in terms of open covers and collections of closed sets with finite intersection property. Compact sets and their properties. Relation between the properties of being closed and compact. Compactness and continuous maps. Compactness of a closed segment. Compactness in Euclidean space. Sequential compactness and its relation to compactness.

Examples of quotient spaces. Möbius strip, Klein bottle, basic surfaces. Descriptions of the real projective space $\mathbb{R}P^n$ and their equivalences.

Product of topological spaces. Topological properties of projections and fibers. Relations between topological properties of topological spaces and their product.