

Present a complete solution for each problem. Answers alone will give no credit.

1. Show all points x on the complex plane such that

- (1) $z^{-1} = \bar{z}$;
- (2) $|\arg z| < \frac{\pi}{6}$;
- (3) $1 \leq |z - 2i| < 2$;
- (4) $1 < |\operatorname{Im} z| \leq 2$;
- (5) $|z - 2| = \operatorname{Re} z + 2$;
- (6) $|z - 1| - |z + 1| = 1$.

2. Let $n \in \mathbb{N}$. Simplify the expression $(1 + \cos \theta + i \sin \theta)^n$ and give the answer in the algebraic form.

3. Prove that $|z + w|^2 + |z - w|^2 = 2|z|^2 + 2|w|^2$ for any $z, w \in \mathbb{C}$ and give a geometrical interpretation of this identity.

4. Prove that complex n th roots of unity form a commutative group with respect to multiplication.