Algebra for Teachers   Name _____

Homework 3

Due 2/26   Score _____

Present a complete solution for each problem. Answers alone will give no credit.

1. In which positional system the following is true: $216 \cdot 3 = 654$?

2. Let G be a group. Prove that for all $a, b, c \in G, \; ac = bc$ implies $a = b$.

3. Let $M = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, b \in \mathbb{R} \right\}$ be the set of 2×2 diagonal matrices. Is M a group with respect to matrix addition? Is M a group with respect to matrix multiplication? Explain!

4. Let $R = \{a + b\sqrt{2} \mid a, b \in \mathbb{Z}\}$ and $S = \{a + b\sqrt[3]{2} \mid a, b \in \mathbb{Z}\}$. Are R, S rings with respect to usual addition and multiplication? Explain!

5. Prove the following properties of a ring R:

a) $a \cdot 0 = 0 \cdot a = 0$ for any $a \in R$

b) $a(-b) = (-a)b = -(ab)$ for any $a, b \in R$.

(In your proofs, cite axioms of a ring you use.)