Stony Brook University
Mathematics Department
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Introduction to Analysis
MAT 320
October 29, 2016

Program for the second midterm.

The topics listed here are recommended for review. In the exam, it may be required to state the relevant definitions and theorems or use them in solutions of problems.

Some of the topics printed in **boldface** will be included (in a rephrased form) in the exam. It will be required to formulate the relevant definitions and theorems, and provide a detailed proofs.

(1) Monotone sequences. Convergence of monotone sequences. Theorem 10.2.
(2) **Lemma on decreasing sequence of closed intervals.**
(3) Cauchy sequences and their convergence. Theorem 10.11.
(4) Subsequences of a sequence. **Existence of a monotonic subsequence** (Theorem 11.4 from the textbook).
(5) **Bolzano-Weierstrass Theorem** (Theorem 11.5 from the textbook).
(6) Lim sup and lim inf.
(7) Topological spaces. **Axioms in terms of neighborhoods and open sets and their equivalence.** Read remarks to the lecture of October 13.
(8) Interior, exterior and boundary points of a set in a topological space. Section 2.1 of the Complements.
(9) Definitions of metric and metric spaces. Balls and spheres in a metric space. Section 2.4 of the Complements and Definition 13.1 in the textbook.
(10) Metric topology. **Theorem 2.1 from the Complements,** 13.6, 13.7 and 13.8 in the textbook.
(11) Topology of a subspace. Section 2.5 from the Complements.
(12) Definition of continuous maps between topological spaces and their simplest properties. Section 3.1 from the Complements.
(13) **Continuity at a point and its relation to continuity.** Section 3.2 from the Complements.
(14) **Sequential continuity and its relation to Continuity.** Section 3.3 from the Complements and Theorem 17.2 from the textbook.
(15) Theorems about operations with continuous functions. Theorems 17.3 and 17.4 from the textbook.
(16) Theorem on sequential continuity of composition of sequentually continuous maps. Theorem 17.5 from the textbook.
(17) **Extreme Value Theorem** (Theorem 18.1 from the textbook).
(18) Sequential compactness. Sequentially compact sets in a metric space are bounded and closed. The converse statement for subsets of Euclidean space. Continuous image of a sequentially compact space is sequentually compact.
(19) **Intermediate Value Theorem** (Theorem 18.2 from the textbook).
(20) Connected spaces. Properties of connected sets. Connected components of a topological space. Theorem about continuous image of a connected space.
(21) Theorems about continuity of monotone functions. (Theorems 18.4-18.6 from the textbook.)