Stony Brook University
Mathematics Department
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Introduction to Analysis
MAT 320
December 7, 2016

Program for the final exam.

The topics listed here are recommended for review. In the exam, it may be required to state the relevant definitions and theorems or use them in solutions of problems.

Some of the topics printed in **boldface** will be included (in a rephrased form) in the exam. It will be required to formulate the relevant definitions and theorems, and provide a detailed proofs.

(1) Properties of the field of real numbers. Axiomatic characterization of it. Completeness axiom 4.4, Archimedean property, Denseness of $\mathbb{Q}$ in $\mathbb{R}$.
(2) Limit of a sequence of real numbers (definition 7.1) Bounded sets on $\mathbb{R}$. A convergent sequence is bounded (theorem 9.1) Limit theorems for sequences (9.2-7). Convergence of a sequence to infinity.
(3) Monotone sequences. **Convergence of bounded monotone sequences. Cauchy criterion for convergence of a sequence.** Theorem 10.11.
(4) Subsequences of a sequence and their limits. Existence of monotonic subsequences (theorems 11.4 and 11.7). Bolzano-Weierstrass theorem.
(5) Definition of a topological space. Open sets, topological structure in a set. Section 2.1 of the Complements.
(6) Neighborhoods of a point in a topological space. Section 2.1 of the Complements.
(7) Interior, exterior and boundary points of a set in a topological space. Section 2.1 of the Complements.
(8) Definitions of metric and metric spaces. Balls and spheres in a metric space. Section 2.4 of the Complements and Definition 13.1 in the textbook.
(9) Metric topology. Theorem 2.1 from the Complements, 13.6, 13.7 and 13.8 in the textbook.
(10) Topology of a subspace. Section 2.5 from the Complements.
(11) **Definition of continuous maps between topological spaces and their simplest properties.** Section 3.1 from the Complements.
(12) **Continuity at a point and its relation to continuity.** Section 3.2 from the Complements.
(13) **Sequential continuity and its relation to continuity.** Section 3.3 from the Complements and Theorem 17.2 from the textbook.
(14) Theorems about operations with continuous functions. Theorems 17.3 and 17.4 from the textbook.
(15) Theorem on sequential continuity of composition of sequentually continuous maps. Theorem 17.5 from the textbook.
(16) Extreme Value Theorem (18.1) and Intermediate Value Theorem (18.2). Theorems about continuity of monotone functions. (18.4-18.6)
(17) Connected topological spaces. Connected sets and their properties. Partition of a topological space into connected components. Connected sets on the line. **Theorem on continuous image of a connected set. Generalized Intermediate Value Theorem.**
(18) **Definition of uniform continuity (19.1). Uniform continuity of a continuous function on a closed interval (19.2)**

(19) Series, convergence and divergence of series, the sum (Section 17.1). Geometric series, condition of its convergence, the sum (Example 1 in 17.1).
(20) Cauchy criterion for convergence of series (14.3-14.5).
(21) **Tests for convergence of series. Comparison test (14.6). Absolute convergence (14.2 and 14.7). Ratio test (14.8) and root test (14.9) with proofs.** Harmonic series and its divergence (15 Example 1). Convergence of $\sum \frac{1}{n^p}$ (15.1). Integral tests (15.2). Alternating series theorem 15.3.
(22) **The Riemann rearrangement theorem.** A series which converges, but does not converge absolutely is called *conditionally convergent*. The Riemann rearrangement theorem states that for any real number M terms of a conditionally convergent series can be permutted so that the sum would become M. See Definitions, Statement of the theorem and Proof sections in the wikipaedia article

`https://en.wikipedia.org/wiki/Riemann_series_theorem`

(23) Invariance of the sum of a positive convergent series under permutations of its terms.
(24) **Convergence radius of a power series** (23.1).
(25) Uniform convergence of a sequence of functions (definition 24.2)
(26) **Continuity of the uniform limit of a sequence of continuous functions (24.3)**
(27) Continuity of power series (26.1 and 26.2).
(28) Theorem about limit of integrals and integral of the limit (25.2)
(29) Definitions of derivative (28.1). Continuity of a differentiable function (theorem 28.2).
(30) Rules for calculation of derivatives (28.3, 28.4)
(31) Vanishing of the derivative at a local extremum of a differentiable function (29.1), Rolle's theorem (29.2) and Mean Value Theorem (29.3). Values of the derivative and behavior of the function (29.4-29.7).
(32) The Darboux integral of a bounded function (definition 32.1). **The test for integrability (theorem 32.5 with lemmas 32.2-32.4).**
(33) **Integrability of monotonic and continuous functions (theorem 33.1 and 33.2).**
(34) Properties of Darboux integrals (33.3-6), Intermediate Value theorem for integrals (33.9).
(35) **Fundamental theorem of Calculus (34.1, 34.3)**
(36) Integration by parts (34.2), and theorem on change of variable (34.4).
(37) Term-wise differentiation and integration of power series.
(38) Taylor series for a function.