MAT 310, Linear Algebra

Name ______________________________

Homework 6

Score____________

1. Let T and S be operators in $\mathcal{P}_3(\mathbb{C})$ defined by $Tp(z) = p(z-1)$, $Sp(z) = p(1-z)$.
(a) Find eigenvalues and eigenvectors of T and S.
(b) Find subspaces invariant under T and invariant under S.

2. Let T be a linear operator over $\mathbb{R}$, let $p \in \mathcal{P}(\mathbb{R})$ such that $p(T) = 0$ and $\lambda \in \mathbb{R}$ be an eigenvalue of T. Is it true that $p(\lambda) = 0$? Prove or find a counter-example.

3. Let T be a linear operator over $\mathbb{R}$ such that $T^2 = \mathrm{id}$. Prove that T is diagonalizable.

4. Let T be a linear operator over $\mathbb{F}$ such that $T^3 = \mathrm{id}$. Prove that
(a) if $\mathbb{F} = \mathbb{C}$ then T is diagonalizable
(b) if $\mathbb{F} = \mathbb{R}$ and T is diagonalizable, then $T = \mathrm{id}$.

5. Let T be the operator on $\mathcal{P}_2(\mathbb{R})$ defined by formula $Tp(x) = (x-1)p'(x)$. Does there exists an operator $S \in \mathcal{L}(\mathcal{P}_2(\mathbb{R}))$ such that $S \neq p(T)$ for any polynomial p?