

Homework 5

due 3/2

Score _____

1. Given a point P inside a triangle ABC . What masses should be placed to A , B and C so that the center of these masses would be P ?
2. Prove that in a triangle ABC a bisector AD divides the side BC so that

$$\frac{|BD|}{|DC|} = \frac{|AB|}{|AC|}$$

3. What masses should be placed to points A , B and C so that the barycenter of them would be the center of the circle inscribed in ABC ?

4. Let A , B , C be points, p, q, r be positive real numbers. Prove that there exists a point D such that for any points O_1 and O_2

$$\begin{aligned}\overrightarrow{O_1D} &= \frac{p}{p+q+r}\overrightarrow{O_1A} + \frac{q}{p+q+r}\overrightarrow{O_1B} + \frac{r}{p+q+r}\overrightarrow{O_1C} \\ \overrightarrow{O_2D} &= \frac{p}{p+q+r}\overrightarrow{O_2A} + \frac{q}{p+q+r}\overrightarrow{O_2B} + \frac{r}{p+q+r}\overrightarrow{O_2C}\end{aligned}$$

What is the meaning of this statement? What is its relation to the notion of barycenter?

5. Let A , B , C be points, p, q, r be positive real numbers. Prove that there exists a point D such that

$$\frac{p}{p+q+r}\overrightarrow{DA} + \frac{q}{p+q+r}\overrightarrow{DB} + \frac{r}{p+q+r}\overrightarrow{DC} = 0.$$

Prove that such D is unique. Hints: 1. How to find D ? 2. See the preceding problem.

6. Generalize the problems 4 and 5 to any finite set of points.