Stony Brook University
Mathematics Department
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MAT 220
Vector Geometry and Algebra
Spring 2017

Practice Midterm 1

This Practice Midterm is given to show what kind of problems you may expect on the exam. Solutions will **not** be given. Why? In order not to keep you busy with reading the specific solutions. The best way to effectively prepare for the exam is to read your lecture notes and the other materials, rather than the solutions. Make sure that you understand what you read. Revisit your quizzes and homeworks. When you will feel comfortable with the material, do the practice problems. Actual Midterm 1 will be shorter and easier.

1. Calculate $(\sqrt{3}+i)^{30}$.

2. Find the complex number $\dfrac{(3-4i)(2-i)}{2+i} - \dfrac{(3+4i)(2+i)}{2-i}$.

(b) Explain how to find the real part of this complex number without calculations.

(c) How does the result of (b) allow to shorten the calculation of the imaginary part?

3. Prove that complex number z and w are conjugate if and only if $z+w$ and zw are real.

4. Solve equations
(a) $\overline{z} = 2 - z$,
(b) $z^2 = \overline{z}$,
(c) $z^2 + (6+i)z + 5 + 5i = 0$.

5. Find $\left(\dfrac{4}{\sqrt{3}+i}\right)^{12}$

6. What is the set of complex numbers such that
(a) $\arg z = \dfrac{\pi}{4}$,
(b) $\arg(z-2+i) = \dfrac{\pi}{4}$,
(c) $\dfrac{\pi}{4} \le \arg(z-2+i) \le \pi$,
(d) $|z-1+2i| \ge 3$,
(e) $\begin{cases} |z-i| \le 1 \\ |z+i| \le 1, \end{cases}$

(f) $|z-1| = |z+2i|$?
Show these sets on pictures.

7. Let M, N, K be the midpoints of the sides of a triangle ABC. Prove that the centroids of ABC and MNK coincide.

8. Find the coordinates of the barycenter of points $A(1,4)$, $B(-3,-2)$, $C(4,-5)$ with the masses of 60, 40 and 80 g respectively.

9. Let O be a point inside a triangle ABC and M be the point where the line AO meets BC. Let $|AO| = 6$, $|OM| = 2$, $|BM| = 5$, $|MC| = 2$. Find the masses to be placed at the vertices of the triangle ABC such that O will be the barycenter.

10. Given a triangle $A(-3,-1)\,B(3,2)\,C(4,-3)$ and a point $P(1,0)$ inside it. Find the masses to be placed at the vertices of the triangle such that P will be the barycenter of the triangle.

11. Prove that the area of a triangle ABC can be found by the formula

$$\frac{1}{2}\sqrt{\overrightarrow{AB}^2 \cdot \overrightarrow{AC}^2 - (\overrightarrow{AB} \cdot \overrightarrow{AC})^2}$$

12. Let $ABCD$ be a trapezoid with bases BC and AD. Prove that

$$|AC|^2 + |BD|^2 = |AB|^2 + |CD|^2 + 2|BC| \cdot |AD|.$$

13. In a pyramid with a square base all edges have the same length. Find the angle between skew medians of two lateral faces.

14. A quadrilateral has vertices $A(3,-2)$, $B(3,5)$, $C(0,4)$, $D(-1,-1)$. Find the coordinates of the point of intersection of the diagonals of the quadrilateral.

15. Prove Leibniz's theorem for a triangle:

If M is the centroid of a triangle $A_1A_2A_3$ and O is an arbitrary point, then

$$\sum_{i=1}^{3} |OA_i|^2 = \sum_{i=1}^{3} |MA_i|^2 + 3|OM|^2.$$

Generalize and prove this theorem for the case of a tetrahedron.