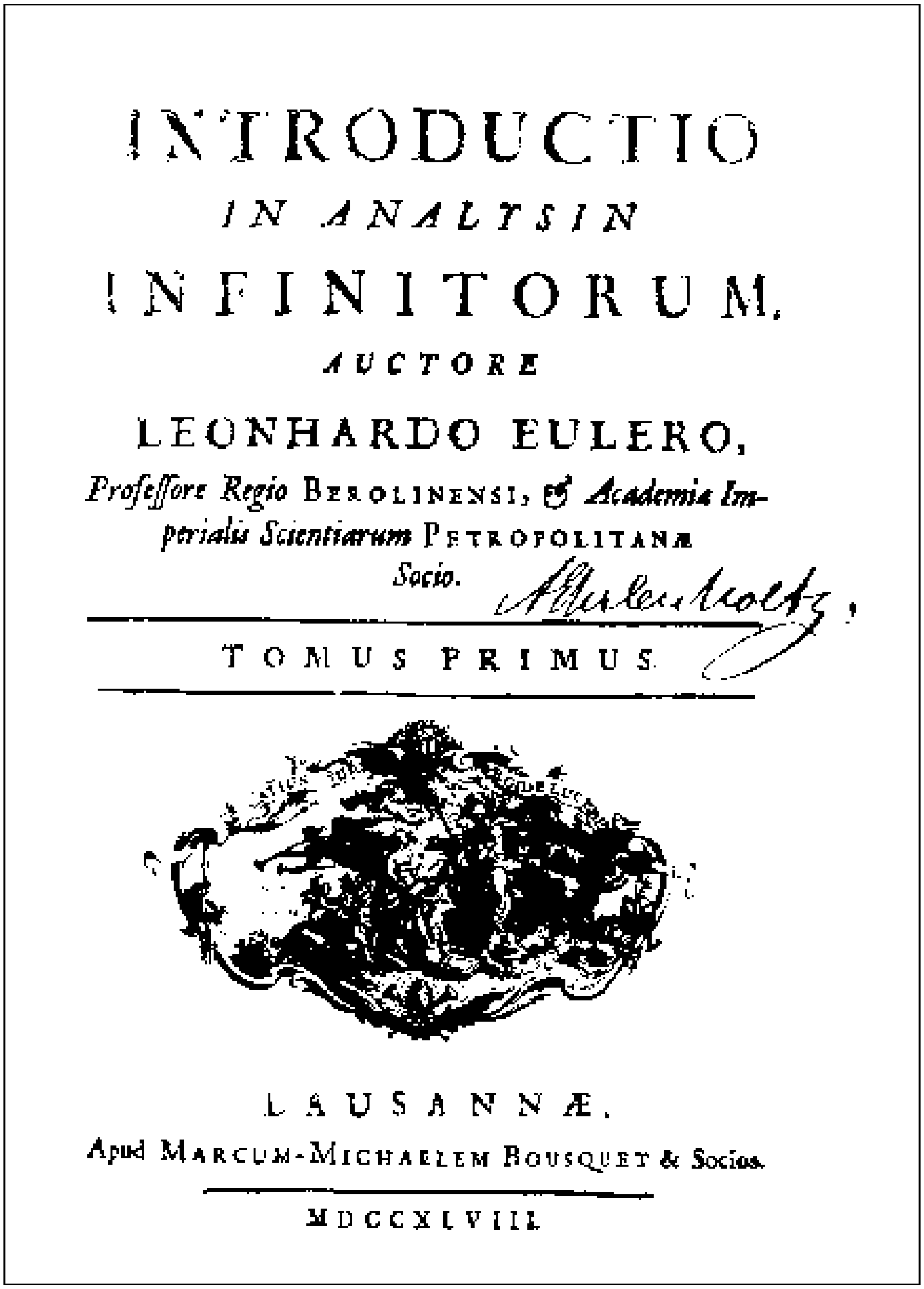

INTRODUCTIO
IN ANALYSIN
INFINITORUM,
AUCTORE
LEONHARDO EULERO,
Professore Regio BEROLINENSI, & *Academiæ Imperialis Scientiarum* PETROPOLITANÆ
Socio.

TOMUS PRIMUS.

LAUSANNÆ,
Apud MARCUM-MICHAELEM BOUSQUET & Socios.

MDCCXLVIII.

Euler's *Introductio in Analysin Infinitorum*, 1748.

Preface

This book represents an introductory course of Calculus. The course evolved from the lectures, which the author had given in the Kolmogorov School in years 1986–1998 for the one-year stream. The Kolmogorov School is a special physics-mathematical undergraduate school for gifted children. Most of the graduates of Kolmogorov School continue their education in Moscow University, where they have to learn the Calculus from the beginning.

This motivates the author efforts to create a course of Calculus, which on the one hand facilitates to the students the perception of the standard one, but on the other hand misses the maximum possible of the standard material to provide the freshness of perception of the customary course. In the present form the course was given in Uppsala University in the autumn semester of 2001 for a group of advanced first-year students.

The material of the course covers the standard Calculus of the first-year, covers the essential part of the standard course of the complex Calculus, in particular, it includes the theory of residues. Moreover it contains an essential part of the theory of finite differences. Such topics presented here as Newton interpolation formula, Bernoulli polynomials, Gamma-function and Euler-Maclaurin summation formula one usually learns only beyond the common programs of a mathematical faculty. And the last lecture of the course is devoted to divergent series—a subject unfamiliar to the most of modern mathematicians.

The presence of a number of material exceeding the bounds of the standard course is accompanied with the absence of some of "inevitable" topics[1] and concepts. There is no a theory of real numbers. There is no theory of the integral neither Riemann nor Lebesgue. The present course even does not contain the Cauchy criterion of convergence. Such achievements of the ninetieth century as *uniform convergence* and *uniform continuity* are avoided. Nevertheless the level of rigor in the book is modern. In the first chapter the greek principle of exhaustion works instead of the theory of limits.

"Less words, more actions" this is the motto of the present course. Under "words" we mean "concepts and definitions" and under "actions" we mean "calculations and formulas". Every lecture gives a new recipe for the evaluation of series or integrals and is equipped with problems for independent solution. More difficult problems are marked with an asterisk. The course has a lot to do with the *Concrete Mathematics* of Graham, Knuth, Patashnik.[2]

The order of exposition in the course is far from the standard one. The standard modern course of Calculus starts with sequences and their limits. This course, following to Euler's *Introductio in Analysin Infinitorum*,[3] starts with series. The introduction of the concept of the limits is delayed up to tenth lecture. The Newton-Leibniz formula appears after all elementary integrals are already evaluated. And power series for elementary functions are obtained without help of Taylor series.

[1]A.Ya. Hinchin wrote: "The modern course of Calculus has to begin with the theory of real numbers".

[2]R.L. Graham, D.E. Knuth, O. Patashnik, *Concrete Mathematics*, Addison-Wesley, 1994.

[3]L. Euler, *Introductio in Analysin Infinitorum*, 1748. Available in *Opera Omnia*, Series I, Volume 8, Springer, 1922.

The course demonstrates the unity of real, complex and discrete Calculus. For example, complex numbers immediately after their introduction are applied to evaluate a real series.

Two persons play a crucial role in appearance of these lectures. These are Alexandre Rusakov and Oleg Viro. Alexandre Rusakov several years was an assistent of the author in the Kolmogorov School, he had written the first conspectus of the course and forced the author to publish it. Oleg Viro has invited the author to Uppsala University. Many hours the author and Oleg spent in "correcting of English" in these lectures. But his influence on this course is far more then a simple correction of English. This is Oleg who convinces the author not to construct the integral, and simply reduces it to the concept of the area. The realization of this idea ascending to Oleg's teacher Rokhlin is one of characteristic features of the course.

The main motivation of the author was to present the power and the beauty of the Calculus. The author understand that this course is somewhere difficult, but he believes that it is nowhere tiresome. The course gives a new approach to exposition of Calculus, which may be interesting for students as well as for teachers. Moreover it may be interesting for mathematicians as a "mathematical roman".