MAT 331-Fall 20: Correction Homework 2

Exercice 1. *Consider a sequence* (S_n) *given by the conditions* $S_0 = S_1 = 1$ *and the recursive formula:*

$$S_n = 3S_{n-1} + 2S_{n-2}, \tag{1}$$

for all $n \geq 2$ *.*

(a) *(1 point) Write a recursive function* ***rec_ sequence(n)*** *which takes* n *and returns the value* S_n*. (test your code by printing* S_0, S_1, S_2, S_3, S_4*)*

(b) *(1 point) Using a* ***for loop****, write a function ,* ***for_ sequence(n)*** *which takes* n *and returns the value* S_n*. (test your code by printing* S_0, S_1, S_2, S_3, S_4*)*

(c) *(2 points) Estimate the memory consumption and complexity for each of these codes. (Explain carefully your computations)*

Correction of question (c). Let us denote by m_n the memory consumption of rec_sequence(n) and by c_n the complexity of that function.

The inductive relation for the function $m(n)$.

$$m_{n+1} = m_n + m_{n-1} + 48. \tag{2}$$

and $m_0 = m_1 = 1$. Denote $u_n = m_n - l$, the inductive relation induced by u_n is given by:

$$u_{n+1} + l = u_n + l + u_{n-1} + l + 48. \tag{3}$$

Setting $l = -48$, we obtain:

$$u_{n+1} = u_n + u_{n-1}, \tag{4}$$

where $u_0 = u_1 = m_0 - l = 1 + 48 = 49$.

This is closely related to the Fibonacci sequence (a sequence defined by a linear recurrence of order 2), which grows like:

$$u_n = O\left((\frac{1+\sqrt{5}}{2})^n \right). \tag{5}$$

This is because the golden mean is the largest root of the polynomial $X^2 - X - 1$. In particular, $m_n = o(r^n)$ where $r = (1+\sqrt{5})/2$.

The computation of the complexity is similar, we compute the number of comparison, addition,multiplications, it satisfies:

$$c_{n+1} = c_n + c_{n-1} + 4, \tag{6}$$

where $c_0, c_1 = 1$. Setting $v_n = c_n - L$, we have:

$$v_{n+1} + L = v_n + L + v_{n-1} + L + 4 \tag{7}$$

Setting $L = -4$, we get:

$$v_{n+1} = v_n + v_{n-1}, \tag{8}$$

so $v_n = O(r^n)$ and $c_n = O(r^n)$ where $r = (1+\sqrt{5})/2$ is the golden mean.

We now compute the complexity c'_n and memory consumption m'_n of the function for_sequence(n).

For the complexity, we do one comparison at the start, initialize three values and execute the code in the loop $n-1$ times. In each loop, we do 2 multiplications, one addition and three assignments, in total:

$$c'_n = 4 + (n-1)(2+1+3) = 6n - 2 = O(n). \tag{9}$$

For the memory consumption, we only work with the variables u,v,value and p. In total $m'_n = 4 \cdot 24 = 96$ bytes (the size of an int is 24 bytes). □