

Probability II

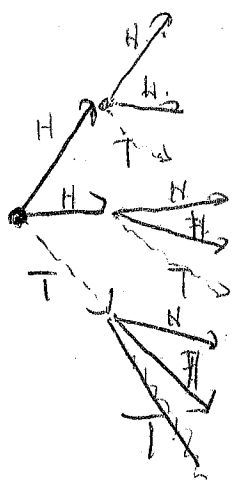
II 1/

Toss an unbalanced coin twice $\left(\frac{2}{3}\right)$ is the probability of getting H $\left(\frac{1}{3}\right)$ P(H, T)

A = get two heads

B = obtain twice the same result

color



9 ends of the tree.

$$HH \rightarrow \frac{4}{9}$$

$$HT \rightarrow \frac{2}{9}$$

$$TH \rightarrow \frac{2}{9}$$

$$TT \rightarrow \frac{1}{9}$$

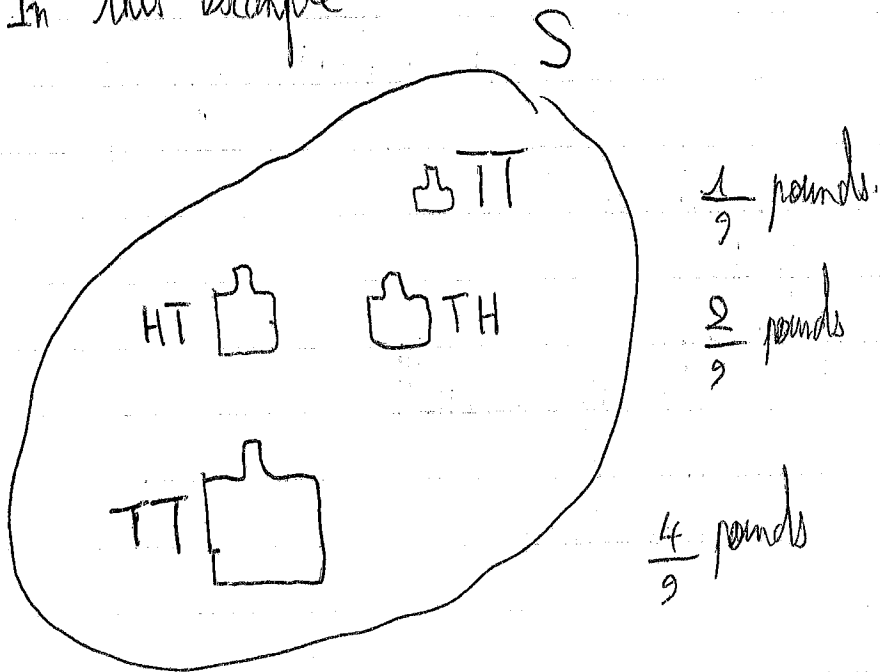
$$P(\text{get two heads}) = P(HH) = \frac{4}{9}$$

$$P(B) = P(\text{twice the same result}) = P(\{HH, TT\})$$

$$= \frac{4}{9} + \frac{1}{9}$$

$$P(B) = \frac{5}{9}$$

In this example



Def : Event = subset of the sample space.

In the double coin toss

"get two heads" $\{HH\}$

"get twice the same result" $\{HH, TT\}$

Simple events

$\{HH\}, \{HT\}, \{TH\}, \{TT\}$

Impossible events $\{\}$

with 2 simple events: $\{HH, HT\}, \{HH, TH\}, \{HH, TT\}, \{HT, TH\},$
 $\{HT, TT\}, \{TH, HH\}.$

With 3 events, $\{HH, HT, TH\}$

$\{HH, HT, TT\}$

$\{HH, TH, TT\}$

$\{HT, TH, TT\}$

Certain event = $\{HH, HT, TH, TT\} = S.$

Solve a probability pt

↳ define a sample space

↳ count the number of elt on that space (simple case)

↳ describe the probability of each simple event

Ques: Today I am going to talk about the first two elements.

$X = \{x_1, x_2, \dots, x_p\}$ p elements

$Y = \{y_1, y_2, \dots, y_q\}$ q elements.

how many elements of the form (x, y) where $x = x_i$
for some i, j are there? $y = y_j$

p pair in $X \times Y$.

Answer $p \times q$.

Application I toss a coin (perfectly balanced) 4 times.

What is the probability that I get only heads?

* Sample space?

* #S?

an element of the sample space
is a 4 tuple of elements which are H or T.

$$\#S = 2 \times 2 \times 2 \times 2 = 16.$$

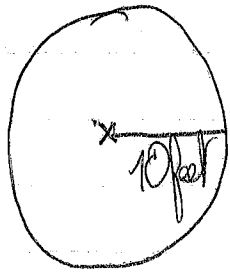
Probability to get only heads $\frac{1}{16}$.

⚠ the tuple is ordered $(H, H, T, T) \neq (T, H, H, T)$.

• when the sample space is infinite!

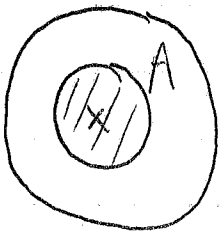
$\frac{\pi}{3}$

I close my eyes and throw a dart on a target



$S =$ pts in the disc of radius 10

what the probability that I hit a pt at distance ≤ 1 feet.



$$P(A) = \frac{\text{Area}(A)}{\text{Area}(\text{disc of radius } 10)} = \frac{\pi 1^2}{\pi 10^2} = \frac{1}{100}$$

$A = \{ \text{pts in the disc of radius } 1 \}$

Single dice roll $S = \{1, 2, 3, 4, 5, 6\}$

Double dice roll

$S?$

$S?$

What is the probability that the sum of the result of the rolls is larger or equal than 10?

↳

$$S = \{ \underbrace{\begin{matrix} \downarrow \\ 1 \end{matrix}}_{1}, \underbrace{\begin{matrix} \downarrow \\ 2 \end{matrix}}_{2}, \underbrace{\begin{matrix} \downarrow \\ 3 \end{matrix}}_{3}, \underbrace{\begin{matrix} \downarrow \\ 4 \end{matrix}}_{4}, \underbrace{\begin{matrix} \downarrow \\ 5 \end{matrix}}_{5}, \underbrace{\begin{matrix} \downarrow \\ 6 \end{matrix}}_{6} \}$$

$$\#S = 6 \times 6 = 36$$

$$A = \{ \text{sum} \geq 10 \}$$

$$A = \{ (6, 4), (5, 5), (4, 6), (6, 5), (5, 6), (6, 6) \}$$

$$P(A) = \frac{7}{36}$$

Choosing cards.

What is a combination?

• Ordered tuple.

I take out 5 cards out of a 54 ~~to~~ cards deck.

