MAT 200, Logic, Language and Proof, Fall 2015 — Homework 9

Due in Class : November 4, 2015.

Reading : Read Chap.15 and Chap.16

Turn in the following exercises.

Problem 1.

Prove that there does not exist a rational number whose square is 10.

Problem 2.

Prove that the set of polynomials with integer coefficients is denumerable. Deduce that the set of algebraic numbers is denumerable.

Hint : You can take for granted the fact that a polynomial equation

$$a_0 + a_1 x + \dots a_n x^n = 0$$

has at most n real solutions.

Problem 3.

Prove that if an integer n is the sum of two perfect squares ($n = a^2 + b^2$ for $a, b \in \mathbb{Z}$), then n is of the form $4q + r$ for some $q \in \mathbb{Z}$, where $r = 0$, $r = 1$ or $r = 2$. Deduce that 1234567 cannot be written as the sum of two squares.