MAT 200, Logic, Language and Proof, Fall 2015 — Homework 4

Due in Class : September 23, 2015.

Turn in the following exercices.

Problem 1.
By using a truth table, prove that, for sets A, B and C,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Illustrate the proof with a Venn diagram.

Problem 2.
Prove that for sets A, B and C, $A \cap B = A \cap C$ and $A \cup B = A \cup C$ if and only if $B = C$.

Problem 3.
Prove or disprove each of the following statements.

(1) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, xy \geq 0$;
(2) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, (x + y > 0 \text{ or } x + y = 0)$;
(3) $(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y > 0)$ and $(\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x + y = 0)$.

Problem 4.
Suppose that $A \subseteq \mathbb{Z}$. Write the following statement and its negative entirely in symbols using the quantifiers $\forall$ and $\exists$:

There is a greatest number in the set A.

Give an example of a set A for which the statement is true, and another example for which it is false.

Problem 5.
Prove that, for sets A, B, C and D,

(1) $A \times (B \cup C) = (A \times B) \cup (A \times C)$;
(2) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.