MAT 200, Logic, Language and Proof, Fall 2015 Homework 1

Due in Class : September 2, 2015.

Reading : Read p.3–29.

Turn in the following exercices.

Problem 1. Explain why the sentence '*This sentence is false*' is not a proposition.

Problem 2. By using truth tables, prove that, for all statements P and Q, the statement '$P \Rightarrow Q$' and its *contrapositive* '$(\text{not}\, Q) \Rightarrow (\text{not}\, P)$' are equivalent.

Problem 3. By using truth tables, prove that, for all statements P and Q, the following statements are equivalent :

(1) '$P \Rightarrow Q$',
(2) '$(P \,\text{or}\, Q) \Leftrightarrow Q$',
(3) '$(P \,\text{and}\, Q) \Leftrightarrow P$'.

Problem 4. Use the properties of addition and multiplication of real numbers given in Properties 2.3.1 to deduce that, for all real numbers a and b,

(1) $a \times 0 = 0 = 0 \times a$,
(2) $(-a)b = -(ab) = a(-b)$,
(3) $(-a)(-b) = ab$.

Recall that for any given real number x, the element $-x$ is the unique real number such that $x + (-x) = 0$.