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Midterm II
MAT 211 Section 3
Spring 2008

Show all your work

Your name: Student ID:

Number	*Score*	*Weight*
1		15
2		15
3		20
4 *ChooseOne*		15
5		15
6		15
7*TakeHome DueThursday*		10
Total		105

Number 7 is a 10 pts Take home -webwork.

The following inverses might be useful for you.

$$\begin{bmatrix} 0 & 1 & 1/3 \\ -1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1/3 & -1/3 \\ 0 & 1 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 1/3 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1/3 & -1/3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(1) (15 pts) Consider a basis $\mathcal{B}$ of R^3.

$$\mathcal{B} = \left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ 1 \\ 0 \end{bmatrix} \right)$$

(a) Find $\vec{v}$, where $[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$.

(b) Find the $\mathcal{B}-$coordinate of $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(2) (15 pts) Consider a transformation $T : R^3 \to R^3$ defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} -x+y+z \\ 2y+z \\ z \end{bmatrix}$$

(a) Find the matrix of the transformation with respect to the standard basis $\mathcal{S}$.

(b) What is (Nullity of T + Rank of T)?

(c) Find the matrix of transformation with respect to $\mathcal{B}-$ basis, where let $\mathcal{B}$ as in (1),

$$\mathcal{B} = \left(\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ 1 \\ 0 \end{bmatrix}\right)$$

(3) (20pts) Let P_2 be the set of all polynomials with degree less than or equal to 2. (degree ≤ 2)and let $T : P_2 \to P_2$ be defined by $T(f) = 3f'$.

(a) Find the matrix of transformation with respect to the standard basis $\mathcal{S} = \{1, x, x^2\}$.

(b) Find the basis of kernel of T

(c) Show that T is linear transformation

(d) Is T an isomorphism? Justify your answer.

(4) (15 pts) Do one of the two

(a) Prove or disprove that

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 0 & 1 \end{bmatrix}$$

are linearly independent.

(b) Prove or disprove that $\{A \in R^{2x2} | det(A) = 1\}$ is a subspace of R^{2x2}

(5) (15 pts) Find the determinant of the following matrices.

(a) $$Det\left(\begin{bmatrix} 0 & 2 & 1 & 0 \\ -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 \end{bmatrix}\right)$$

(b) $$Det\left(\begin{bmatrix} 2 & 2 & -780 & 99 \\ 0 & 1 & 101 & -1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)$$

(c) Find $$det\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{11} & a_{12} & a_{13} & a_{14} \end{bmatrix}\right)$$

(6) (15 pts) Let $T : R^4 \to R$ defined by

$$T(\vec{x}) = det\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ x_1 & x_2 & x_3 & x_4 \end{bmatrix}\right), \text{ where } \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

Suppose that

$$T(\left(\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}\right) = det\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 1 & 2 & 3 & 4 \end{bmatrix}\right) = 3, \;\; T(\left(\begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \end{bmatrix}\right) = 4$$

(a) Find $T(\begin{bmatrix} 11 \\ 12 \\ 13 \\ 14 \end{bmatrix})$

(b) Find $T\left(\begin{bmatrix} 20 \\ 20 \\ 20 \\ 20 \end{bmatrix}\right)$

(7) (10 pts) Take Home. Due Date Thursday April 17 noon.
Let

$$A = \begin{bmatrix} 0 & 1 & 1/3 \\ -1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(a) Solve $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ using Cramer's Rule.

(b) Use Adjoint as in Fact 6.3.10 to find A^{-1}