Spring2008-MAT211 1

The following is the open end version of Midterm1Preptest in the webwork. If you can handle the following you will score at least 80/

1.
$$\begin{array}{rcrcl} -6x & + & 15y & = & 6 \\ -14x & + & 35y & = & k \end{array}$$

Find the value of k , for the above system of equations to be consistent.

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2. If $A = \begin{bmatrix} 3 & -1 & -1 \\ -3 & 3 & 2 \\ -2 & -2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 & -4 \\ 1 & 3 & -4 \\ 4 & 2 & 3 \end{bmatrix}$

Find $4A - 3B$
and $2A^T$

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3. Solve the system

$$\begin{cases} x_1 + x_2 + 4x_3 = -9 \\ 4x_1 + 3x_2 + 3x_3 = 5 \end{cases}$$

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4. The dot product of two vectors $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$ in $\mathbb{R}^n$ is defined by $x \cdot y = x_1y_1 + x_2y_2 + \ldots + x_ny_n$.
The vectors x and y are called perpendicular if $x \cdot y = 0$.
Then a vector in $\mathbb{R}^3$ perpendicular to $\begin{bmatrix} -9 \\ 8 \\ 3 \end{bmatrix}$ can be written as a linear combination of two vectors. Express those vectors in $\mathbb{R}^3$ perpendicular to $\begin{bmatrix} -9 \\ 8 \\ 3 \end{bmatrix}$ as a linear combination of two vectors.

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5. The reduced row-echelon forms of the augmented matrices of four systems are given below. How many solutions does each system have?

1. $\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 16 \\ 0 & 0 & 0 \end{array}\right]$
   - A. Infinitely many solutions
   - B. Unique solution
   - C. No solutions
   - D. None of the above
2. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & -13 \end{array}\right]$
   - A. Unique solution
   - B. No solutions
   - C. Infinitely many solutions
   - D. None of the above
3. $\left[\begin{array}{ccc|c} 1 & 0 & 15 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$
   - A. No solutions
   - B. Infinitely many solutions
   - C. Unique solution
   - D. None of the above
4. $\left[\begin{array}{ccc|c} 1 & 0 & -7 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$
   - A. Infinitely many solutions
   - B. No solutions
   - C. Unique solution
   - D. None of the above

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6. Determine the value of k for which the system

$$\begin{cases} x + y + 4z = 3 \\ x + 2y - 2z = -1 \\ 3x + 9y + kz = -14 \end{cases}$$

has no solutions.
Remark: The question can be posted as a matrix form also.

7.
To see if $b = \begin{bmatrix} -7 \\ 1 \\ -3 \end{bmatrix}$ is a linear combination of the vectors $a_1 = \begin{bmatrix} -3 \\ -2 \\ -5 \end{bmatrix}$ and $a_2 = \begin{bmatrix} 6 \\ 10 \\ -8 \end{bmatrix}$ one can solve the matrix equation $Ax = c$. Find A and c

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8. Write the system

$$\begin{cases} -6y + 3z = -3 \\ 4x - 7y = -4 \\ -2x + 9y - 5z = 2 \end{cases}$$

in matrix form.

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9. Consider a linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^2$ for which
$T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$, $T\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$,
and $T\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$.
Find the matrix A of T.

10. Find the matrix A of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ given by

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \\ 6 \end{bmatrix} x_1 + \begin{bmatrix} -3 \\ 5 \\ -1 \end{bmatrix} x_2.$$

11. Consider the orthogonal projection onto the line L in $\mathbb{R}^2$ that consists of all scalar multiples of the vector $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$. I.e. for a given input $\begin{bmatrix} x \\ y \end{bmatrix}$, find the orthogonal projection onto the line in the direction of $\begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Note that it is a linear transformation from $R^2 to R^2$.

12.) Let $A = \begin{bmatrix} -4 & 9 \\ 2 & 9 \\ 2 & -6 \end{bmatrix}$.

Define the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ **as** $T(x) = Ax$.

Find the images of $u = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ **and** $v = \begin{bmatrix} a \\ b \end{bmatrix}$ **under** T **.**

13. Determine which of the following transformations are linear transformations. If it is not a linear transformation, give a reason why it is not.

- **A. The transformation** T **defined by** $T(x_1, x_2, x_3) = (x_1, 0, x_3)$
- **B. The transformation** T **defined by** $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$.
- **C. The transformation** T **defined by** $T(x_1, x_2, x_3) = (x_1, x_2, -x_3)$
- **D. The transformation** T **defined by** $T(x_1, x_2, x_3) = (1, x_2, x_3)$
- **E. The transformation** T **defined by** $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$.

14. Find a linearly independent set of vectors that spans the same subspace of $\mathbb{R}^3$ **as that spanned by the vectors**

$$\begin{bmatrix} -3 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix}.$$

Find the linearly independent set of vectors.
Find the basis of the subspace spanned by the above three vectors.

15.Let $A = \begin{bmatrix} 12 & 16 & -4 \\ 6 & 8 & -2 \end{bmatrix}$.

Find bases of the kernel and image of A **(or the linear transformation** $T(x) = Ax$**).**

16. Solve the equation $-8x_1 + 3x_2 + 8x_3 - 9x_4 = 0$ **and Find a basis of the subspace of** $\mathbb{R}^4$ **defined by the equation** $-8x_1 + 3x_2 + 8x_3 - 9x_4 = 0$.

17. Let W_1 **be the set:** $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Determine if W_1 **is a basis for** $\mathbb{R}^3$ **and check the correct answer(s) below.**

- **A.** W_1 **is a basis.**
- **B.** W_1 **is not a basis because it does not span** $\mathbb{R}^3$**.**
- **C.** W_1 **is not a basis because it is linearly dependent.**

Let W_2 **be the set:** $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Determine if W_2 **is a basis for** $\mathbb{R}^3$ **and check the correct answer(s) below.**

- **A.** W_2 **is not a basis because it is linearly dependent.**
- **B.** W_2 **is a basis.**
- **C.** W_2 **is not a basis because it does not span** $\mathbb{R}^3$**.**

18. Let

Let $A = \begin{bmatrix} 1 & 4 & 5 & -2 \\ 0 & 2 & -2 & -2 \\ -3 & -16 & -11 & 10 \end{bmatrix}$ **and** $b = \begin{bmatrix} 1 \\ 3 \\ -9 \end{bmatrix}$.

[?] **1. Is** b **in the image of** A**?. Support your answer.**

19. (1 pt) Library/TCNJ/TCNJ_NullColumnSpaces-/problem9.pg

A **is an** $m \times n$ **matrix.**
Check the true statements below and give a brief reason supporting your answer:

- **A. The kernel of a linear transformation is a vector space.**
- **B. The null space of an** $m \times n$ **matrix is in** $\mathbb{R}^m$**.**
- **C.** $ColA$ **is the set of all vectors that can be written as** Ax **for some** x**.**
- **D. If the equation** $Ax = b$ **is consistent, then** $ColA$ **is** $\mathbb{R}^m$**.**
- **E. The null space of** A **is the solution set of the equation** $Ax = 0$**.**
- **F. The column space of** A **is the range of the mapping** $x \to Ax$**.**

20. Find the projection of $v = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$ **onto the line** l **of** $\mathbb{R}^3$ **given by the parametric equation** $l = tu$**, where** $u = \begin{bmatrix} -1 \\ -5 \\ 4 \end{bmatrix}$