MAT 200 OUTLINE, PART 2

I'm preparing a lecture outline for the benefit of those who are unable to make it to class due to illness or other reasons. See the course textbook for additional details about most of these items. If a theorem is listed as **Theorem.**, this means that you should be familiar with the proof.

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- Definition of a function
- Ways of representing and visualizing a function (a formula, a graph, a diagram, a table of values)
- Domain, codomain, image of a function
- Equality of functions: two functions are equal if they have the same domain and codomain and are equal at every point
- Restriction of a function to a subset A, denoted by $f|A$
- Composition of two functions (associative but not commutative)

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- Sequence: a function whose domain is $\mathbb{N}$, may be defined by a formula or defined inductively
- Limit of a sequence, null sequence
- The graph of a function: the set $\{(x, f(x) : x \in X\}$
- Injective, surjective, bijective functions. Note that these properties depend very much on the choice of domain and codomain.
- The inclusion function $i\colon X \to Y$, where $X \subset Y$; the projection function $p_1 : X \times Y \to X$ or $p_2\colon X \times Y \to Y$

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- Review of Midterm 1
- Introduction to LaTeX. Don't forget to do the LaTeX assignment or wait until the last moment! Talk to me if you have any questions or need help getting started.

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- Invertibility of a function f, the inverse function f^{-1}
- **Theorem.** f is invertible if and only if f is a bijection. In this case, the inverse of f is unique.
- Defining inverses of functions like sin, cos, tan by restricting domain and range

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- The functions $\overrightarrow{f} : \mathcal{P}(X) \to \mathcal{P}(Y)$ and $\overleftarrow{f} : \mathcal{P}(Y) \to \mathcal{P}(X)$. Note that these are always defined.
- Cardinality of a finite set: X has cardinality n if there is a bijection $f\colon \mathbb{N}_n \to X$.
- Theorem. Suppose $f\colon \mathbb{N}_m \to X$ and $g\colon \mathbb{N}_n \to X$ are bijections. Then $m = n$.

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- The addition principle: If X, Y are disjoint and finite sets, then $|X \cup Y| = |X| + |Y|$.
- The multiplication principle: If X and Y are finite sets, then $|X \times Y| = |X| \cdot |Y|$.
- **Theorem.** The inclusion–exclusion principle: If X, Y are finite sets, then $|X \cup Y| = |X| + |Y| + |X \cap Y|$.

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- (Proofs of some of the earlier results.)
- The pigeonhole principle: Suppose that $f : X \to Y$ is a function between finite sets such that $|X| > |Y|$. Then f is not an injection: there exist distinct points $x, y \in X$ such that $f(x) = f(y)$.
- Example: In any group of two of more people, there are two people with exactly the same number of friends in the group. (Assume that friendship is reflexive.)

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- maximum and minimum of a finite set of real numbers
- the set of divisors of an integer
- the greatest common divisor of two integers; coprime/relatively prime integers
- **Theorem.** An integer a is odd if and only if $a = 2q + 1$ for some $q \in \mathbb{Z}$.

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- The set $\mathrm{Fun}(X, Y)$ of functions from X to Y. (Also denoted by Y^X.)
- **Theorem.** For any finite sets X, Y, $|\mathrm{Fun}(X, Y)| = |Y|^{|X|}$.
- The set $\mathrm{Inj}(X, Y)$ of injections from X to Y.
- Theorem. Suppose that X, Y are finite sets and $m = |X|$ and $n = |Y|$, $m \le n$. Then $|\mathrm{Inj}(X, Y)| = n(n-1)\cdots(n-m+1) = n!/(n-m)!$. This value is also called the falling factorial $(n)_m$.
- a permutation of X
- an r-subset of X, the binomial coefficient $\binom{n}{r}$
- Pascal's triangle

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- Example: how many 5-card hands of playing cards? How many full houses?
- Example: expanding the binomial $(a + b)^n$
- Theorem. For all $n, r \in \mathbb{N}$ satisfying $1 \le r \le n$, $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$
- **Theorem.** For all $n, r \in \mathbb{Z}^{\geq}$ satisfying $r \le n$, $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$.