Stony Brook University
Department of Mathematics

MAT 200– Exam 1
Logic, Language, and Proof

Instructor: Dr. Eun Hye Lee

March 5, 2020

Name: _____
Solar ID Number: _____

This exam contains 8 pages (including this cover page) and 7 questions. Total of points is 100.

Please show **all** your work! Answers without supporting work will not be given credit unless otherwise stated. Write answers in spaces provided.

No calculator is allowed for this exam.

You have 80 minutes to complete this exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	12	
5	13	
6	15	
7	30	
Total:	100	

1. (10 points) Using the truth table, show that $P \vee \neg Q$ is equivalent to $\neg(\neg P \wedge Q) \vee (P \wedge \neg R)$.

2. (10 points) Prove that for all positive integer n, $n^2 + 3n + 2$ is even.

3. (10 points) Show that for any sets A, B, and C, $A \cup (B - C) = (A \cup B) - (C - A)$, using set operations.

4. Let $A = \{1, 2, 3\}$, $B = \{a, b\}$, and $C = \{\pi, 2, b\}$. List the elements of the sets below:
   (a) (4 points) $\mathcal{P}(A)$: the power set of A;
   (b) (4 points) $(A \cup B) - C$; and
   (c) (4 points) $A \times (B \cup C)$.

5. Consider a function $f(x) = x^2 - 1$ from different domains to $\mathbb{R}$.
   (a) (3 points) If the domain of f is $[1, \infty)$, find the image of f. Write your answer using *interval notations*.
   (b) (6 points) If the domain of f is $[-2, 0)$, compute $f(-2)$, $f(-1)$, and $f(0)$.
   (c) (4 points) What is the largest image of f you can have within real numbers? Write your answer using *interval notations*.

6. (15 points) Define functions f and $g : \mathbb{R} \to \mathbb{R}$ by:

$$f(x) = x^2 \qquad \text{and} \qquad g(x) = x^2 - 1.$$

List the elements of the set

$$\{x \in \mathbb{R} | f \circ g(x) = g \circ f(x)\}.$$

7. (30 points) **True or False.** Indicate your choice clearly by circling either **T** or **F**. A correct answer is worth 3 points, a blank answer is worth 1 point, and an incorrect answer is worth -1 point. You do not have to justify your answer.

   (a) **T** **F** $(n = 2 \text{ or } n = -1) \iff n^2 - n - 2 = 0.$

   (b) **T** **F** $(n = 2 \text{ and } n = -1) \implies n^2 - n - 2 = 0.$

   (c) **T** **F** $\{x^2 | x \in \mathbb{Q}\} = \mathbb{Q}.$

   (d) **T** **F** $\{2x | x \in \mathbb{Q}\} = \mathbb{Q}.$

   (e) **T** **F** $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ x + y = 0.$

   (f) **T** **F** $\exists y \in \mathbb{R}, \forall x \in \mathbb{R},\ x + y = 0.$

   (g) **T** **F** $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ xy = 0.$

   (h) **T** **F** $\exists y \in \mathbb{R}, \forall x \in \mathbb{R},\ xy = 0.$

   (i) **T** **F** $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ xy = 1.$

   (j) **T** **F** $\exists y \in \mathbb{R}, \forall x \in \mathbb{R},\ xy = 1.$