MAT 203: CALCULUS III WITH APPLICATIONS

Final Exam – May 10, 2018

Spring Semester

Name, Lastname:

ID Number:

Recitation Section: Mon Tue Wed

Directions: The exam starts at 5.30pm and ends at 8:00pm. The exam consists of 8 problems. Calculators and notes are not allowed. Show all relevant work in order to get full credit. In case you need to use the restroom, let the instructor know about this. The back of each page can be used as scratch paper.

Scores	
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Problem 1. Consider the function $f(x, y) = x^2 - xy - y^2 - 3x - y$. Find the critical points of $f(x, y)$ and classify them as relative maxima, minima, or saddle points.

Problem 2. A wire in the xy-plane has the shape of a curve C parametrized by

$$\vec{\mathbf{r}}(t) = \cos(t)\,\hat{\mathbf{i}} + \sin(t)\,\hat{\mathbf{j}}$$

with $0 \leq t \leq \frac{\pi}{2}$. The density of the wire is given by the function $\rho(x, y) = x + y$. Calculate the total mass of the wire.

Problem 3. (1). Check that the vector field $\vec{\mathbf{F}}(x,y) = -e^x\big(\sin(y)\,\hat{\mathbf{i}} + \cos(y)\,\hat{\mathbf{j}}\big)$ is conservative in the xy-plane.
(2). Find a potential of $\vec{\mathbf{F}}$.
(3). Calculate the work done by $\vec{\mathbf{F}}$ on a particle that is moving in the plane from the point $(0,\pi)$ to the point $(1,2\pi)$ along a straight line.

Problem 4. Consider the surface S of equation $z = xy$.

(1). Write the equation of the tangent plane to S at the point $(x_0, y_0) = (2, 3)$.

(2). Calculate the area of the surface $z = xy$ over the region $x^2 + y^2 \leq 1$.

Problem 5. (1). Calculate the volume of the solid bounded by

$$0 \le y \le 1, \quad 0 \le x \le y, \quad 0 \le z \le xy.$$

(2). Calculate the volume of the solid in the space bounded by the surface $z = 6 - x^2 - y^2$, and the xy-plane defined by the equation $z = 0$.

Problem 6. Evaluate the integral

$$\int\int_R (2x + 2y)\, dA$$

where R is the plane region bounded by the lines $y - x = -1$, $y - x = 2$, $y + x = -1$, and $y + x = 3$. Use the change of variables $x = \frac{v-u}{2}$ and $y = \frac{v+u}{2}$, or any other suitable change of variables at your choice.

Problem 7. (1). Evaluate the integral

$$\int_0^1 \int_x^1 e^{y^2}\, dy\, dx$$

(2). Evaluate the following limit

$$\lim_{n\to+\infty} \int_0^1 \int_0^1 x^n y^n\, dx\, dy$$

Problem 8. (1). Use Green's theorem in order to calculate the work done by the force field

$$\vec{\mathbf{F}}(x,y) = xy\,\hat{\mathbf{i}} + (x+y)\,\hat{\mathbf{j}}$$

on a particle moving counterclockwise around the closed path C defined by $x^2 + y^2 = 1$.
(2). Say whether $\vec{\mathbf{F}}$ is conservative or not, and provide a short justification.