Exam 4 — Fall 2019

1. $\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{9}\le 1 \qquad -1\le z\le 1$

Outer boundary: $z = 3\sqrt{1-\frac{x^2}{4}-\frac{y^2}{4}} = 3\sqrt{1-\frac{r^2}{4}}$

$z=1 \Longrightarrow \frac{r^2}{4} = 8/9 \Rightarrow r = \frac{4\sqrt{2}}{3}$

Use cylindrical coordinates:

$$2\int_0^{2\pi}\int_0^{\frac{4\sqrt{2}}{3}}\int_0^1 r\,dz\,dr\,d\theta$$

(spherical coordinates also possible)

$$+2\int_0^{2\pi}\int_{4\sqrt{2}/3}^{2}\int_0^{3\sqrt{1-r^2/4}} r\,dz\,dr\,d\theta$$

$$= 2(2\pi)\frac{1}{2}r^2\Big|_0^{4\sqrt{2}/3} + 2(2\pi)\int_{4\sqrt{2}/3}^{2} 3\sqrt{1-r^2/4}\; r\,dr$$

$$= 2\pi\frac{32}{9} + 4\pi\; 3\left(1-r^2/4\right)^{3/2}\frac{2}{3}(-2)\Big|_{4\sqrt{2}/3}^{2} = \frac{64\pi}{9} + 16\pi\left(1-\frac{32}{9\cdot 4}\right)^{3/2}$$

$$= \frac{64\pi}{9} + 16\pi\,(1/9)^{3/2} = \frac{64\pi}{9} + \frac{16\pi}{27}$$

$$= \boxed{\frac{208\pi}{27}}$$

2. (a) $\vec{F} = \langle x, 1\rangle$

$\text{curl}\,\vec{F} = \frac{\partial}{\partial x}(1) - \frac{\partial}{\partial y}(x) = 0-0=0.$

Since $\vec{F}$ is defined on all $\mathbb{R}^2$, which is simply connected, $\vec{F}$ is conservative.

(b) $f_x = x \Longrightarrow f(x,y) = \frac{1}{2}x^2 + h(y)$

$f_y(x,y) = h'(y) = 1 \Longrightarrow h(y) = y$

$f(x,y) = \frac{1}{2}x^2 + y$

$\int_C \vec{F}\cdot d\vec{r} = f(e,1) - f(1,0) = \frac{1}{2}e^2 + 1 - \frac{1}{2} = \boxed{\frac{1}{2}e^2 + \frac{1}{2}}$

(c) $\vec{r}\,'(t) = \langle x, 1\rangle \Longrightarrow \frac{dx}{dt} = x, \quad \frac{dy}{dt} = 1$

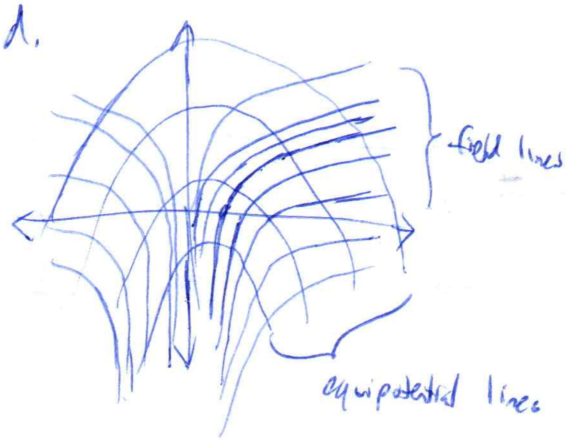
$\Longrightarrow \frac{1}{x}\,dx = dt, \quad dy = dt \Longrightarrow \ln(x) = t + C_1, \quad y = t + C_2$

$(x,y) = \vec{r}(t) = \langle e^{t+C_1}, t + C_2\rangle = \langle c_1 e^t, t + C_2\rangle$

(where $c_1 = e^{C_1}$)

*Note: (c) and (d) have not been covered in our course.

2.



$$f(x,y) = \frac{x^2}{2} + y = C \implies y = C - \frac{x^2}{2}$$

3. Not covered yet.

4. Not covered yet.

5. Not covered yet