Stony Brook University
Mathematics Department
Julia Viro

MAT 203, Lec 02
Calculus III with Applications
Fall 2019

Final Exam

Show all your work, that is provide **complete** solution for the problems. Answers alone will give **no** credit. No textbooks, notes, electronic devices, baggy clothes. No bathroom trips. No wandering in the exam room. By the end of the exam, please remain seated and follow the proctors instructions.

Last name ________________ First name ________________ Student ID #________________

Recitation group ________ Recitation Instructor ________________________

Problem #	Points/Total
1	/6
2a	/3
2b	/3
2c	/3
2d	/3
3a	/3
3b	/5
3c	/3
3d	/5
4	/8
5	/8
Total	/50

R20 W 11:00pm-11:53pm Physics P130 Myeongjae Lee
R22 F 12pm-12:53pm Physics P127 Juan Ysimura
R23 Tu 4:00pm- 4:53pm Library W4530 Juan Ysimura
R24 Th 2:30pm- 3:23pm Physics P130 Siquing Zhang

Problem 1. Find the volume of the wheel of cheese

$$\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{9} \leq 1, \quad -1 \leq z \leq 1.$$

Problem 2. Consider the plane vector field $\mathbf{F} = (x, 1)$.

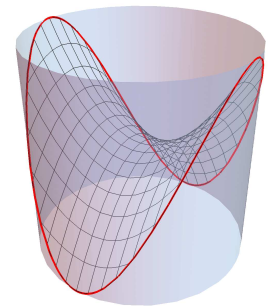
a) Show that $\mathbf{F}$ is conservative and find its potential.

b) Calculate the work of the field $\mathbf{F}$ along the curve $y = \ln x$ from $x = 1$ to $x = e$.

Problem 2 (cont.)

c) Find the equation of the field lines for $\mathbf{F} = (x, 1)$.

d) Draw the field lines and equipotential lines on the same coordinate system.



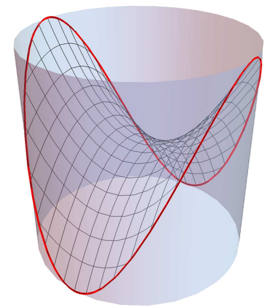
Problem 3.

Consider the saddle surface $z = x^2 - y^2$ and cylinder $x^2 + y^2 = 1$.

Let S be the part of the saddle surface $z = x^2 - y^2$
that is situated inside the cylinder $x^2 + y^2 = 1$.

a) Find a paramertization of S.

b) Find the area of S.



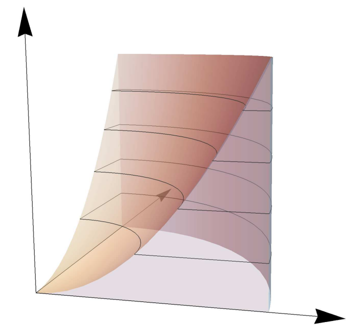
Problem 3 (cont.)

Consider the saddle surface $z = x^2 - y^2$ and cylinder $x^2 + y^2 = 1$.

Let C be the intersection curve of $z = x^2 - y^2$ and $x^2 + y^2 = 1$.

c) Find a paramertization of C.

d) Find the circulation of $\mathbf{F} = (z, 2x^2y, y^2)$ along C oriented counterclockwise as seen from above.



Problem 4. Let V be the solid that is situated in the first octant and bounded by the paraboloid $z = x^2 + y^2$, cylinder $x^2 + y^2 = 1$, and coordinate planes. Calculate the flux of $\mathbf{F} = (xz, x^2y, y^2z)$ outward through the boundary of V.

Problem 5. Use Green's formula to prove that the area of a region bounded by a simple closed curve C is equal to

$$\frac{1}{2}\int_C -y\,dx + x\,dy.$$

Use this formula to calculate the area of the elliptic disk $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \leq 1$.

	Line Integrals	Surface Integrals
scalar field f	$\boxed{\int_C f(x,y,z)\,ds}$ $ds = \|\mathbf{r}'(t)\|\,dt$ $\mathbf{r}(t)$ is a parametrization of C Length of $C = \int_C ds$	$\boxed{\iint_S f(x,y,z)\,dS}$ $dS = \|\mathbf{r'}_u \times \mathbf{r'}_v\|\,dudv$ $\mathbf{r}(u,v)$ is a parametrization of S Area of $S = \iint_S dS$
vector field F	$\boxed{\int_C \mathbf{F}\cdot d\mathbf{r}}$ work or circulation (if C is closed) $\mathbf{r}$ is a parametrization of C	$\boxed{\iint_S \mathbf{F}\cdot\mathbf{N}\,dS}$ flux $\mathbf{N}\,dS = (\mathbf{r'}_u \times \mathbf{r'}_v)\,dudv$ $\mathbf{r}(u,v)$ is a parametrization of S

Divergence theorem: $$\oiint_{S=\partial V} \mathbf{F}\cdot\mathbf{N}\,dS = \iiint_V \operatorname{div}\mathbf{F}\,dV$$

Stokes' theorem: $$\oint_{C=\partial S} \mathbf{F}\cdot d\mathbf{r} = \iint_S \operatorname{curl}\mathbf{F}\cdot\mathbf{N}\,dS$$

Green's theorem: $$\oint_{C=\partial D} Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy$$