Exam 1 — Spring 2019

1. $3x^2 + 2xyz + z^2 = 11$

$\nabla F(x,y,z) = \langle 6x + 2yz,\ 2xz,\ 2xy + 2z \rangle$

$\nabla F(1,1,2) = \langle 6+4,\ 4,\ 2+4 \rangle = \langle 10, 4, 6 \rangle$.

$10(x-1) + 4(y-1) + 6(z-2) = 0$

$\boxed{10x + 4y + 6z = 26}$

$\vec{r}(t) = \langle 1, 1, 2 \rangle + t\langle 10, 4, 6 \rangle$

$\boxed{\vec{r}(t) = \langle 1+10t,\ 1+4t,\ 2+6t \rangle}$

2. (Midterm 1)

3. (Midterm 1)

4. $f_x(x,y) = 3x^2 - 2x = 0$, $f_y(x,y) = 2y = 0$

$\Rightarrow x(3x-2) = 0$ $\Rightarrow y = 0$

$\Rightarrow x = 0,\ 2/3$ Critical points: $(0,0)$, $(2/3, 0)$

$f_{xx} = 6x - 2$

$f_{xy} = 0$

$f_{yy} = 2$

$$d = \begin{vmatrix} 6x-2 & 0 \\ 0 & 2 \end{vmatrix} = 12x - 4$$

$d(0,0) = -4 < 0 \Rightarrow$ $\boxed{(0,0) \text{ is saddle point}}$

$d(2/3, 0) = 8 - 4 = 4 > 0$, $f_{xx}(2/3, 0) > 0 \Rightarrow$ $\boxed{(2/3, 0) \text{ is a local minimum}}$

5. $\nabla f(x,y) = \langle 4x + 3,\ 2y \rangle$ $\quad g(x,y) = x^2 + y^2$

$\nabla g(x,y) = \langle 2x, 2y \rangle$

$$\begin{cases} 4x + 3 = \lambda(2x) \\ 2y = \lambda(2y) \\ x^2 + y^2 = 4 \end{cases}$$

$\Rightarrow \lambda = 1$ or $y = 0$

$(\lambda = 1) \Rightarrow 4x + 3 = 2x \Rightarrow 2x = -3$

$x = -3/2$

$\Rightarrow 9/4 + y^2 = 4$

$y^2 = 4 - 9/4 = \frac{16-9}{4} = \frac{7}{4}$

$\Rightarrow y = \pm \frac{\sqrt{7}}{2}$

$y = 0 \Rightarrow x = \pm 2$

$f(2,0) = 8 + 6 + 0 = 14$

$f(-2,0) = 8 - 6 + 0 = 2$

$f\left(-3/2, \frac{\sqrt{7}}{2}\right) = 9/2 - 9/2 + 7/4 = 7/4$

$f\left(-3/2, -\frac{\sqrt{7}}{2}\right) = 9/2 - 9/2 + 7/4 = 7/4$

$\boxed{\begin{array}{l} \text{maximum: } 14 \\ \text{minimum: } 7/4 \end{array}}$