MAT 203: CALCULUS III WITH APPLICATIONS

Midterm 1 – February 23rd, 2018

Spring Semester

Name, Lastname:

ID Number:

Recitation Section: Mon Tue Wed

Directions: The exam starts at 12.00pm and ends at 12:53pm. The exam consists of 5 problems. Calculators and notes are not allowed. Show all relevant work in order to get full credit. In case you need to use the restroom, let the instructor know about this. The back of each page can be used as scratch paper.

Scores	
1	
2	
3	
4	
5	
Total	

Problem 1. Consider the vectors $\vec{\mathbf{u}} = \langle -2, 2, 1 \rangle$ and $\vec{\mathbf{v}} = \langle 3, -1, 1 \rangle$.

(1). Find $\cos(\theta)$ where θ is the angle formed by the vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

(2). Find the area of the parallelogram determined by the vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$.

Problem 2. Consider the points $P(1,0,2)$ and $Q(-1,2,3)$ in the space, and denote by L the line that passes through P and Q.

(1). Find the equation of the plane T that is perpendicular to the line L, and that passes through the point $R(0,1,0)$.

(2). Find the distance between the plane T and the point $S(-1,1,1)$.

(3). Find the distance between the points $R(0,1,0)$ and $S(-1,1,1)$.

Problem 3. This problem consists of two independent subproblems. Solve all of them.
(1). Find the center of the following quadric

$$x^2 - 2y^2 - z^2 - 2x - 6z - 8 = 0.$$

(2). Find the equation of the surface of revolution obtained by revolving the curve $x = 7z^2$ about the x-axis. Sketch the graph of the surface.

Problem 4. An object moves in the plane with acceleration $\vec{\mathbf{a}}(t) = 2\,\hat{\mathbf{i}} + e^t\,\hat{\mathbf{j}}$. Answer to the following questions supposing that at time $t = 0$ the object starts from rest at the point $P(1, 1)$.

(1). Find velocity and position functions of the object.

(2). Find the speed of the object at time $t = 1$.

Problem 5. Consider the following vector-valued function with two components:

$$\vec{\mathbf{r}}(t) \;=\; (t^2-1)\,\hat{\mathbf{i}} \;+\; \big(t(t^2-1)\big)\,\hat{\mathbf{j}}.$$

(1). Find either symmetric or parametric equations of the line that is tangent to the graph of $\vec{\mathbf{r}}(t)$ at $t = 2$.

(2). Find either symmetric or parametric equations of the line that is perpendicular to the tangent line you found in the previous point, and that passes through the point $Q(1, 2)$.