

Midterm 2

Show all your work, that is provide **complete** solution for the problems. Answers alone will give **no** credit. No textbooks, notes, electronic devices, baggy clothes. No bathroom trips. No wandering in the exam room. By the end of the exam, please remain seated and follow the proctors instructions.

Last name _____ First name _____ Student ID # _____

Recitation group _____ Recitation Instructor _____

Problem #	Points/Total
1	/5
2	/5
3	/7
4	/8
5	/8
6	/7
Total	/40

R20 W 11:00pm-11:53pm Physics P130 Myeongjae Lee
R22 F 12pm-12:53pm Physics P127 Juan Ysimura
R23 Tu 4:00pm- 4:53pm Library W4530 Juan Ysimura
R24 Th 2:30pm- 3:23pm Physics P130 Siquing Zhang

Problem 1. Show that the function $z = f(x, y)$, where $x = u - v$ and $y = -u + v$, satisfies the equation

$$\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = 0.$$

Problem 2. Find a linear approximation of the function $f(x, y) = (x^2 + y)^3$ near the point $(1, 0)$.

Problem 3. Find an equation of the tangent plane to the surface $e^{xz} + yz = 2$ at the point $(0, 1, 1)$.

Problem 4. Find all critical points of the function $f(x, y) = x^3 + y^3 - 3xy$ and determine their types (local minimum, local maximum, saddle point).

Problem 5. Using methods of multivariable calculus, find the extreme values of $f(x, y) = 3x - 4y$ under the constraint $x - y^3 = 0$. Draw a picture on the coordinate plane that shows the level curves of f and the constraint. Give geometric description of the extreme points.

Problem 6. A metallic lamina (plate)

$$\{(x, y) \mid 1 \leq x^2 + y^2 \leq 9, 0 \leq y \leq x\}$$

has the surface density $\rho(x, y) = \arctan \frac{y}{x}$. Find the mass of the lamina.