

MAT203 Calculus III. Midterm Exam 1

10/04/2017

Problem	Points
Problem 1(10pts)	
Problem 2(10pts)	
Problem 3(20pts)	
Problem 4(20pts)	
Problem 5(20pts)	
Problem 6(20pts)	
Total (100pts)	

Note: Please write clearly and cleanly, and provide intermediate steps.

You need to show HOW you obtain your answer.

Calculator, cell phone, cheat sheet are not allowed in this exam.

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: _____

Date: _____

Name: (Please print.) _____ **ID #:** _____

TA's Name and Day of Recitation (M, T, W, Th or F) _____

Problem 1

$$\mathbf{u} = (0, -1, 1), \quad \mathbf{v} = (1, 2, -1).$$

(1) [5pts] Let θ be the angle between \mathbf{u} and \mathbf{v} , then compute $\cos \theta$.

(2) [5pts] Compute $\mathbf{u} \times \mathbf{v}$ and $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$.

Problem 2

(1) [5pts] Write down the equation of the surface obtained by rotating the curve $x^2 - 2y^2 = 1$ about the y -axis.

(2) [5pts] Write down the equation for the sphere of radius 4 centered at the point $(-1, 1, 1)$.

Problem 3.

(1) [10pts] Let \mathcal{L} be a line and its equation is

$$\begin{cases} x = 1 - t \\ y = 1 + t \\ z = t. \end{cases} \quad (1)$$

Compute the equation of the plane \mathcal{H} which contains the point $P(0, 1, 2)$ and is **perpendicular** to \mathcal{L} .

(2) [10pts] Compute the distance from the point $P(1, -1, 2)$ to the plane $2x + 3y + z = 0$.

Problem 4. A curve Γ is described by the vector valued function

$$\mathbf{r}(t) = (3 \cos t, 3 \sin t, t^2 + 2).$$

(1)[10pts] Compute the vector of velocity $\mathbf{v}(t) \equiv \mathbf{r}'(t)$ and the acceleration $\mathbf{a}(t) \equiv \mathbf{v}'(t)$.

(2) [10pts] When $t = 1$, compute the **unit** tangent vector $\mathbf{T}(1)$. Calculate the component of acceleration in the direction of \mathbf{T} when $t = 1$, that is, calculate the scalar product $a_T \equiv \mathbf{a}(1) \cdot \mathbf{T}(1)$.

Problem 5. Let Γ be a helix

$$\mathbf{r}(t) = (3 \sin t, 4t + 1, -3 \cos t). \quad (2)$$

(1) [10pts] Compute the velocity $\mathbf{v}(t) = \mathbf{r}'(t)$ and compute the equation of the tangent line of the curve $\mathbf{r}(t)$ at $\mathbf{r}(0) = (0, 1, -3)$.

(2) [10pts] Calculate the arc length for part of Γ from $\mathbf{r}(-\pi)$ to $\mathbf{r}(\pi)$.

Problem 6.[20 pts] Let the curve Γ determined by the equation $\mathbf{r} = \mathbf{r}(t)$ which satisfies the following conditions,

$$\begin{cases} \mathbf{r}''(t) = (2 \cos t, 2 \sin t, 0), \\ \mathbf{r}'(0) = (0, 1, 1) \\ \mathbf{r}(0) = (-1, 0, 0). \end{cases} \quad (3)$$

- (1) [10pts] Explicitly write down the expression of $\mathbf{r}(t)$ in terms of t .
- (2) [10pts] Compute the unit tangent vector $\mathbf{T}(t)$ at $t = \pi$.

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