FINAL EXAM, MAT 203, LECTURE 02: WINTER 2018

Instructions: The use of phones, books, or notes is not allowed during the exam time. Each problem is worth 25 points and the exam is worth 200 points. Please circle or box your answer and show the work which led to your answer.

Print Name:

Recitation number (circle one):

R20 R21 R22

ID Number:

1	2	3	4	5	6	7	8	Total (/200)

Problem 1:

- For $\vec{r}(t) = e^t\vec{i} + \cos(\pi t)\vec{j} + \sin(\pi t)\vec{k}$, compute $\vec{r}\,'(t)$. Find the parametric equation for the line tangent to $\vec{r}$ when $t = 1$.

- Find the length of the curve given by $\vec{v}(t) = \langle e^{t^2}, 2e^{t^2} \rangle$ where t ranges between 0 and 3.

Problem 2: Suppose the temperature of a heated metal plate (placed in the xy-plane) is given by

$$T(x, y) = 200 - 3x^2 + y^2.$$

Suppose a lizard on the plate will always walk in the direction of most rapid increase in heat. If the lizard starts at point $(2, 1)$, find a vector-valued function which represents its path.

Problem 3: Find three non-negative numbers x, y, and z which add up to 30 and so that $x^2 + y^2 + z^2$ is as small as possible. [For full credit, you need to show your answer is correct.]

Problem 4: Find the equation of the tangent plane to the surface given by

$$z^2 - 2x^2 - 2y^2 = 12$$

at the point $(1, -1, 4)$.

Problem 5: Find the volume of the region obtained by drilling a cylindrical hole of radius 1 around the z-axis through the region trapped under the graph of the paraboloid $z = 4 - x^2 - y^2$.

Problem 6: Find the maximum and minimum of the function $f(x, y) = 2x + y$ among all (x, y) satisfying the equation $2x^2 + y^2 = 1$.

Problem 7: Find the center of mass of a planar lamina associated to the quarter of the unit disc in the portion of the xy-plane where $x, y \geq 0$ with density function $\rho(x, y) = x^2 + y^2$.

Problem 8: Compute the work done by the force $\vec{F}(x, y) = \langle 2x - 3y, -3x - y + 4\rangle$ on a particle traveling from $(3, 0)$ to $(-3, 0)$ along the top half of the circle of radius 3 centered at the origin.

Problem 9: [**10 BONUS PTS**] Use Green's theorem to find the area inside the curve parameterized by

$$\vec{r}(t) = \langle \sin(2t), \sin(t) \rangle$$

where $0 \leq t \leq \pi$.