MAT 203

Final Exam. May 20, 2019

This is a closed notes/ closed book/ electronics off exam.

Please write legibly and show your work.

Each problem is worth 20 points.

Full Name:					
Problem	1	2	3	4	5
Grade					
Problem	6	7	8	9	10
Grade					
Total:					

Problem 1. Let S be the surface given by $x^2 + 2y^2 - z^2 = 21$. Find a unit normal vector to the surface at the point $(2, 3, 1)$ and give an equation of the tangent plane through the point.

Problem 2. Let $F(x, y, z) = \ln(x^2 + y^2) + z$.

a. Find the directional derivative $D_{\underline{u}}F(1, 0, 2)$ in the direction $\underline{u} = \left\langle \frac{6}{7}, \frac{3}{7}, \frac{2}{7} \right\rangle$.

b. Determine the direction of greatest increase at the point $(1, 0, 2)$.

Problem 3. Let $F(x,y) = \frac{1}{3}x^3 - xy^2 - x$. Find all critical points of F and determine whether each is a local minimum, local maximum or saddle point.

Problem 4. Find the maximum and minimum of $f(x, y, z) = 2x + 3y + 4z$ on the surface $x^2 + y^2 + z^2 = 1$.

Problem 5. The upper hemisphere $H = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1, z \geq 0\}$ is given mass density $f(x, y, z) = z$.

a. Find the total mass of the solid H.

b. Calculate the center of mass of H.

(Hint: the volume element in spherical coordinates is $dV = \rho^2 \sin\phi d\rho d\theta d\phi$.)

Problem 6. Let H be the surface $H = \{(x, y, 16 - x^2 - y^2) : x^2 + y^2 \leq 16\}$. Find the surface area of H.

Problem 7. Let C be the curve $\underline{r}(t) = \langle t, \frac{e^t+e^{-t}}{2}\rangle$, $0 \leq t \leq 1$.

a. Calculate the unit tangent vector $T(t)$ and the unit normal vector $N(t)$ to the curve. (Hint: $N(t)$ is a 90 degree rotation of $T(t)$.)

b. Calculate the flux of the vector field $F(x, y) = \langle 1, y\rangle$ across C in the upward direction, that is, calculate

$$\int_C F \cdot N ds.$$

Problem 8.

a. Show that the vector field

$$F(x, y) = \langle 2xe^{x^2+y^2}, 2ye^{x^2+y^2} + e^y \rangle$$

is conservative, and calculate a potential function.

b. Let C be a smooth curve oriented to begin at $(0, 0)$ and end at $(2, 3)$. Calculate

$$\int_C F \cdot d\underline{r}.$$

Problem 9. Let C be the boundary of the rectangle $[0,3] \times [2,6]$, oriented in the counter-clockwise direction. Use Green's Theorem to calculate

$$\int_C (\arctan x + ye^{xy})dx + (xe^{xy} + \sin y + x)dy.$$

Problem 10. Let $R = \{(x, y) : 0 \leq x + y \leq 1, 0 \leq x - y \leq 1\}$. Calculate

$$\iint_R e^{4x+2y} dA.$$

Use for scratch.

Use for scratch.

Use for scratch.

Use for scratch.