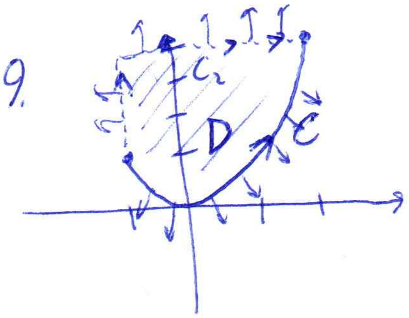
7. $\vec{r}(\theta, z) = \langle 2\cos(\theta), 2\sin(\theta), z\rangle \qquad 0 \le \theta \le 2\pi, \qquad 0 \le z \le 4$

$$\|\vec{r}_\theta \times \vec{r}_z\| = \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2\sin(\theta) & 2\cos(\theta) & 0 \\ 0 & 0 & 1 \end{vmatrix} \right\| = \|\langle 2\cos(\theta), 2\sin(\theta), 0\rangle\| = 2$$

$$\iint_H f(x,y,z)\,dS = \int_0^{2\pi}\int_0^4 \frac{4\cos^2(\theta)\, z}{4^3}\,dz\,d\theta = \frac{1}{16}\int_0^{2\pi}\cos^2(\theta)\,d\theta \cdot \int_0^4 z\,dz = \frac{1}{16}\int_0^{2\pi}\frac{1+\cos(2\theta)}{2}\,d\theta \cdot \frac{1}{2}z^2\Big|_0^4$$

$$= \frac{1}{16}\left[\frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right]_0^{2\pi} \frac{1}{2\cdot 16} = \boxed{\frac{\pi}{2}}$$

9. 

Introduce curve C_2 as drawn on the graph.

Observe $\operatorname{div}\vec{F} = -\sin(1+xy) - xy\cos(1+xy) + \sin(1+xy) + xy\cos(1+xy) = 0$

So
$$\iint_D \operatorname{div}\vec{F}\,dA = 0 = \int_C \vec{F}\cdot\vec{n}\,ds - \int_{C_2}\vec{F}\cdot\vec{n}\,ds$$
by the Divergence form of Green's Theorem.

Now
$$\int_{C_2}\vec{F}\cdot\vec{n}\,ds = \int_1^4 \langle \sin(1-y) - 4\cos(\pi y),\ y\sin(1-y) - 6\rangle \cdot \langle -1, 0\rangle\,dy$$
$$+ \int_{-1}^2 \langle -x\sin(1+4x) - 4\cos(4\pi),\ 4\sin(1+4x) + 6x\rangle \cdot \langle 0, 1\rangle\,dx$$

$$= \int_1^4 4\cos(\pi y) - \sin(1-y)\,dy + \int_{-1}^2 4\sin(1+4x) + 6x\,dx$$

$$= \frac{4}{\pi}\sin(\pi y) + -\cos(1-y)\Big|_1^4 + -\cos(1+4x) + 3x^2\Big|_{-1}^2$$

$$= 1 - \cos(-3) - \cos(9) + 12 + \cos(-3) - 3 = \boxed{10 - \cos(9)} = \int_{C_2}\vec{F}\cdot\vec{n}\,ds = \int_C \vec{F}\cdot\vec{n}\,ds$$