MAT 200 MIDTERM 3 SAMPLE QUESTIONS

1. We say that n is a perfect square if $n = k^2$ for some $k \in \mathbb{Z}$. Prove that if n is a perfect square then $n = 4q$ or $n = 4q + 1$ for some $q \in \mathbb{Z}$.

[You may use the fact that m is an odd number if and only if $m = 2l + 1$ for some $l \in \mathbb{Z}$.]

2. Let $m \geq 1$ be an integer. Prove that if $a \equiv b \mod m$, then for all $c \in \mathbb{Z}$ we have $a + c \equiv b + c \mod m$.

3.

(a) Use the Euclidean algorithm to find the gcd of 23 and 107.
(b) Can you find integers m, n such that $23m + 107n = 1$? If not, why not?
(c) Can you find integers m, n such that $23m + 107n = 3$? If not, why not?
(d) Find all x such that $23x \equiv 3 \mod 107$

4.

(a) State the division theorem and prove its uniqueness part.
(b) Use the Euclidean algorithm to find the greatest common divisor of 136 and 232.
(c) Find integers m, n such that $16 = 136m + 232n$.

5. What is the last digit of 2^{1000}? Explain your answer.

[You may use properties of the congruence modulo some non-zero integer number.]

6. Prove that $7^{19} + 6^{19}$ is divisible by 13.