MIDTERM 2 FORMULAS

Second derivative test: for a function $f(x,y)$ with continuous second derivatives and critical point (a,b), examine the quantity (sometimes called the "Hessian")

$$d = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{yx}(a,b) & f_{yy}(a,b) \end{vmatrix} = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy}(a,b))^2$$

Cylindrical coordinates: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$.
Spherical coordinates: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$.
Integration in polar coordinates example:

$$\iint_R f(x,y)\,dA = \int_a^b \int_{f_1(\theta)}^{f_2(\theta)} f(r\cos(\theta), r\sin(\theta))r\,dr\,d\theta.$$

Mass m of a lamina R with density $\rho(x,y)$: $m = \iint_R \rho(x,y)\,dA$
Moments of mass with respect to x- and y-axis: $M_x = \iint_R y\,\rho(x,y)\,dA$, $M_y = \iint_R x\,\rho(x,y)\,dA$
Center of mass: $(\bar{x}, \bar{y}) = (M_y/m, M_x/m)$
Moment of inertia about the x- and y-axis: $I_x = \iint_R y^2\,\rho(x,y)\,dA$, $I_y = \iint_R x^2\,\rho(x,y)\,dA$
Radius of gyration for object with mass m and moment of inertia I: $\bar{\bar{r}} = \sqrt{I/m}$
Surface area for surface $z = f(x,y)$ over the domain R: $\iint_R \sqrt{1 + f_x(x,y)^2 + f_y(x,y)^2}\,dA$
Integration in cylindrical coordinates example:

$$\iiint_R f(x,y,z)\,dV = \int_a^b \int_{f_1(\theta)}^{f_2(\theta)} \int_{g_1(r,\theta)}^{g_2(r,\theta)} f(r\cos(\theta), r\sin(\theta), z)r\,dz\,dr\,d\theta.$$

Integration in spherical coordinates example:

$$\iiint_R f(x,y,z)\,dV = \int_a^b \int_{f_1(\theta)}^{f_2(\theta)} \int_{g_1(\phi,\theta)}^{g_2(\phi,\theta)} f(\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta.$$

Jacobian for the change of variables transformation $x = g(u,v)$, $y = h(u,v)$: $\left|\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\right|$
Line integral of vector field $\mathbf{F}(x,y,z) = \langle M, N, P\rangle$ on oriented curve C, parametrized by $\mathbf{r}(t)$, $a \le t \le b$:

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C M\,dx + N\,dy + P\,dz = \int_C \mathbf{F}\cdot\mathbf{T}\,ds = \int_a^b \mathbf{F}(x(t), y(t), z(t))\cdot\mathbf{r}'(t)\,dt$$

For the vector field $\mathbf{F} = \langle M, N, P\rangle$, curl $\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$ and div $\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$