MAT 203 Midterm 1
Section 02, Fall 2021

October 6, 2021

Name: ______________________________

Question	Points possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Instructions

1. There are 5 problems on this exam.
2. You have 75 minutes to take the exam.
3. If you need more space, you may continue your solution on the scratch work page and leave a note to direct the grader there.
4. No notes, books, calculators or other resources are allowed.
5. Keep bags, coats and other personal belongings at some distance from your seat.
6. You may ask me for clarification on any question, although I may not be able to answer.

Exam

1. Let $\mathbf{u} = \langle 1, 2, 3\rangle$ and $\mathbf{v} = \langle 2, -3, 1\rangle$.
(a) (4 points) Compute the dot product $\mathbf{u} \cdot \mathbf{v}$ and the cross product $\mathbf{u} \times \mathbf{v}$.

(b) (1 point) The angle θ formed by $\mathbf{u}$ and $\mathbf{v}$ is...

(a) zero ($\theta = 0$)

(b) acute ($0 < \theta < \pi/2$)

(c) right ($\theta = \pi/2$)

(d) obtuse ($\pi/2 < \theta < \pi$)

(e) straight ($\theta = \pi$)

(c) (2 points) What is the area of the triangle whose vertices are the origin and the points $P(1, 2, 3)$ and $Q(2, -3, 1)$?

(d) (2 point) Let $\mathbf{w}$ be the unit vector in $\mathbb{R}^3$ that maximizes the expression $\mathbf{u} \cdot \mathbf{w}$. What is $\mathbf{w}$?

(e) (1 point) How many distinct unit vectors $\mathbf{w}$ in $\mathbb{R}^3$ are there that maximize the expression $\|\mathbf{u} \times \mathbf{w}\|$?

(a) one

(b) two

(c) infinitely many

2. Consider the points $P(1,-1,0)$, $Q(3,1,1)$ and $R(-2,0,1)$ in $\mathbb{R}^3$.
(a) (4 points) Find an equation for the plane S determined by the points P, Q, R.

(b) (2 points) Write a set of parametric equation for the line normal to the plane S in part (a) passing through the point P.

(c) (4 points) Find the distance from the origin to the plane S (that is, the minimum distance from the origin to a point on S).

3. We consider the motion of a particle in $\mathbb{R}^3$ from time $t = 0$ to $t = \pi$ with velocity vector

$$\mathbf{r}'(t) = \langle \sin(t), \cos(t), 1 \rangle.$$

Assume that this particle has initial position $\mathbf{r}(0) = \langle 0, 0, 0 \rangle$.
(a) (3 points) Determine the position function $\mathbf{r}(t)$ for the particle.

(b) (2 points) Compute the unit tangent vector $\mathbf{T}(t)$ for the particle.

(c) (3 points) Compute the (principal) unit normal vector $\mathbf{N}(t)$ for the particle.

(d) (2 points) Find the curvature K of the curve traced by $\mathbf{r}(t)$ at time $t = \pi/2$.

4. Let S be the surface in $\mathbb{R}^3$ defined by the equation

$$x^2 + 2y^2 - z^2 = 4.$$

(a) (4 points) Sketch a graph of S. Make sure you clearly depict the xy-trace, the yz-trace and the xz-trace. (For example, label the points where S intersects one of the three coordinate axes.)

(b) (1 point) S is a(n)...

(a) ellipsoid

(b) hyperboloid of one sheet

(c) hyperboloid of two sheets

(d) elliptic cone

(e) elliptic paraboloid

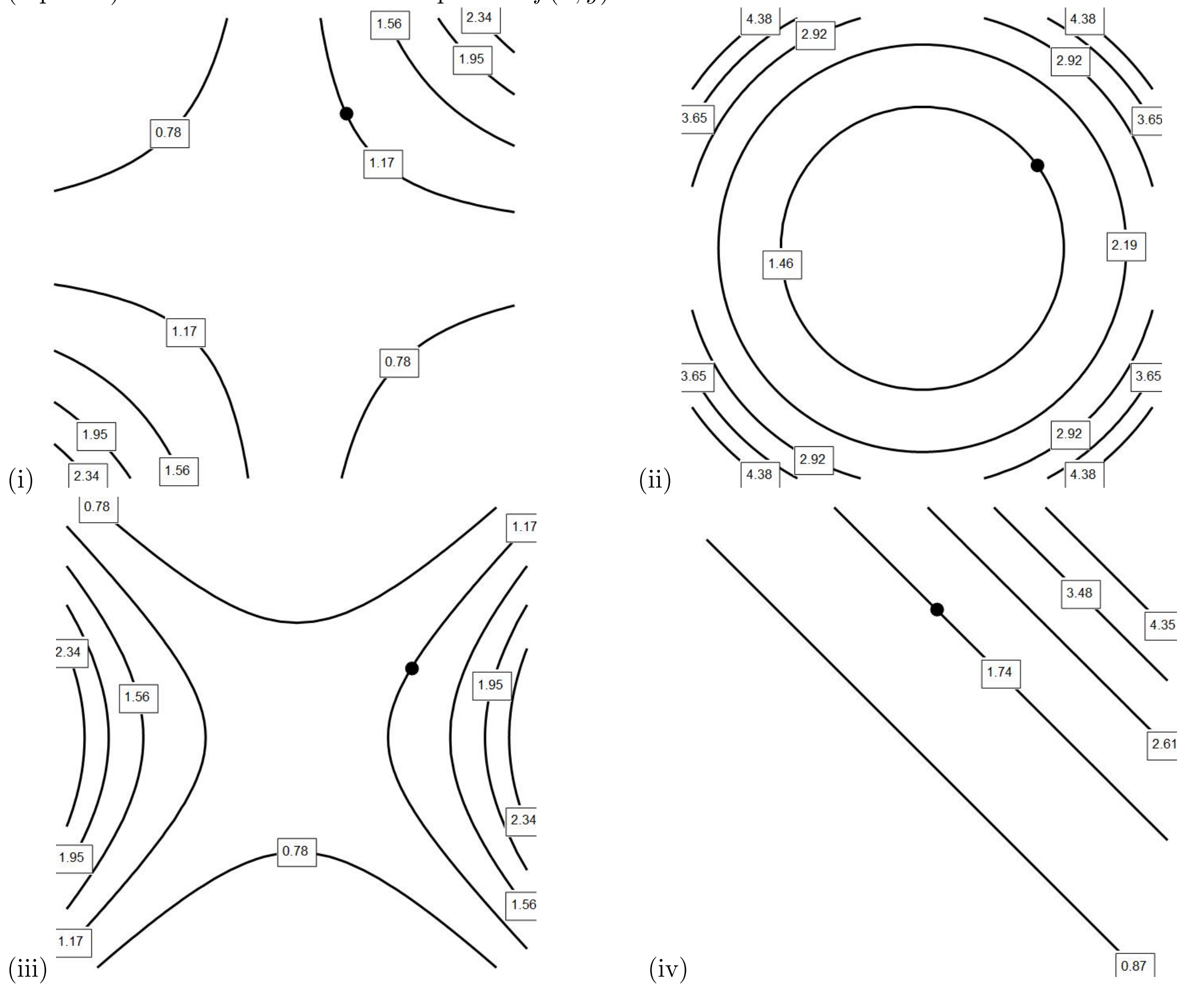
(f) hyperbolic paraboloid

(c) (5 points) The top half of S can be described as the graph of the function $z = f(x, y) = \sqrt{x^2 + 2y^2 - 4}$, defined on the domain $D = \{(x, y) : x^2 + 2y^2 > 4\}$.

Find the linearization of $f(x, y)$ at the point $(2, 1, \sqrt{2})$. Your answer should express z as a function of x and y. (Equivalently, find an equation for the tangent plane to S at the point $(2, 1, \sqrt{2})$).

5. In this problem, we consider the function $z = f(x, y) = e^{x^2-y^2}$, where $-1 \le x \le 1$ and $-1 \le y \le 1$.

(a) (2 points) Select the correct contour plot for $f(x, y)$.



(b) (1 point) Sketch the gradient vector for $f(x, y)$ at the point labeled on the contour plot you selected in part (a).

(c) (3 points) Find the directional derivative of $f(x, y)$ at the point $(x, y) = (1/2, 1/2)$ in the direction $\mathbf{u} = \langle 2/\sqrt{5}, 1/\sqrt{5} \rangle$.

(d) (4 points) Suppose now that x and y are both functions of a variable t, with $x = (t-1)^3$ and $y = \cos(t) + \sin(t)$. Use the chain rule to compute $\dfrac{dz}{dt}$ at the value $t = \pi$.

[SCRATCH WORK]