Tips for learning math

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1. Study at a slow enough pace to internalize what you study. Equally importantly, don't study passively. Scrutinize each proof! Do you agree with each step being carried out? Can you fill in any missing details? Can the argument be reduced down to a small number of simple, intuitive ideas? Can you apply the same idea in other situations? The premise of pure mathematics is that its results are grounded in *rigorous proof*. Thus you should get in the habit of making your own judgments of how convincing a given argument is. At the more advanced level, this will be a hugely important skill. At the beginning level, you're exposed to mainly correct mathematics (although often not completely "rigorous"), but this thought process will help solidify and deepen your understanding.

Don't be afraid to think at a slower pace than your peers. If a classmate seems to grasp something faster than you, there is a high chance that it's just because they've already encountered a similar idea before. Don't confuse a slower pace with a lack of intelligence.

I believe that thoroughness is usually more efficient in the long run. Once you master an idea, it becomes easy and automatic when it appears again in a different context. Mathematics consists of a relatively small number of simple ideas, dressed up in formal jargon, that get applied again and again.

2. Focus on fundamental examples. Many theorems can be viewed in the following way: a certain set of examples represents the general behavior of some abstractly defined class of objects. In the other direction, most theorems require some hypothesis in order for the conclusion to be true. Typically, there is a counterexample to show that the conclusion may fail when the hypothesis is not satisfied. In this way, examples delineate the landscape of mathematical truth.

When you focus on examples, it makes the material more concrete. This makes it more meaningful and comprehensible. From a psychological point of view, a *narrative* is a powerful structure. Consider how, without having made any deliberate prior effort, you can recall the plot of your favorite movie down to the minor details. In a similar way, examples provide the narrative of mathematics. They are a frame on which you can build the house of your understanding.

The essence of math is the complimentary nature of theory (the abstract) and examples (the concrete). Theory without examples is usually incomprehensible. Examples without theory is tedious. Both are "dry", though for different reasons. Math gains its interest from the interaction of the two. Examples suggest theorems, which in turn gives insight into a wider range of examples. Unfortunately, the teaching of mathematics often puts too much weight on the theory. But, in such cases, you can make up the difference with some proactiveness on your part.

You'll find that senior mathematicians can reason very quickly and efficiently about a seemingly wide range of problems. This might seem like a superhuman ability. Quite possibly, they've just mastered their own personal set of key examples. When faced with an abstract proposition, they just run this proposition through the case of these examples.

Another closely related mental structure is to frame things as a *question*. The advantage of a good question is that it has a definite answer. It focuses your mental energy and also serves to make the material concrete. It encourages precision in your thinking. And it provides a measuring stick for your understanding.

3. Don't get bogged down by abstraction. As a student, you'll run into many abstract definitions: topological spaces, metric spaces, manifolds, the various structures of abstract algebra, and so on. In math, a definition is what it is, no more and no less. While it's very good to develop an intuition about these various objects, there's a danger of looking for some deeper meaning that just isn't there. John von Neumann once said, "In mathematics you don't understand things. You just get used to them." I don't advise taking this quote too literally, but I think it reinforces the point of not worrying about understanding everything right away. Be comfortable with an incomplete understanding.

So how should one approach this? Consistent with Tip 2, whenever you encounter an abstract definition, make sure that you have a small number of representative concrete examples in mind. Recognize that the typical theorems we prove about these concrete examples depend only on certain properties they have. So we just make a list of these properties and call it a definition. Whenever the abstract concept is invoked, feel free to replace it in your mind with one of your examples.

To really explore the scope of a definition to its limits—to really understand "what is a topological space" or "what is a finite group", for example—is usually a very non-trivial task. Indeed, really the only way is to become a specialist in that topic and devote years of your life to studying it.

4. Repetition, repetition, repetition! A passage in the textbook might be incomprehensible the first time you read it. Come back a day later, a week later, a month later. You'll often find that the topic in question has gone from incomprehensible to obvious. One important qualification: repetition is not the same as *rote* repetition. Rather, you should emphasize the ideas, intuitions and connections. You should take a proactive approach: with each successive repetition, try to reproduce on your own a larger portion of the material.

5. Get the most out of lecture. In my view, your individual study and the lecture serve distinct roles, even if it's the same material being covered in both. The purpose of the lecture is to focus on the big picture: the main concepts, intuitions, connections. The purpose of individual study is to focus on the details: to really convince yourself that every claim is sound, every line of reasoning works as it should.

For most people, it is virtually impossible to learn math simply by listening to another person lecture. If you don't put the work in outside of class, the lecture will be incomprehensible and stay that way.

At the other extreme, it can be perfectly viable to pick up a textbook, work through it, and learn a subject. However, a pure self-study approach will probably be less efficient and ultimately limiting. There is a human side to the subject that can't be found just in a book.

Consider some analogies. A game of chess can be reduced down to a single paragraph of notation: 1. e4 e5 2. Nf3 Nc6, etc. This notation contains all the information about a game, yet it doesn't capture the essence of playing the game of chess. Or consider that the notes on a musical score carry all the information about a piece of music, again without capturing any of the essence of music. Likewise, the essence of **doing** mathematics is something parallel to, but largely distinct from, what can be read in a book or article.

It's perfectly fine, even expected, that you won't understand every word uttered in a lecture. The goal is that, through lecture, you learn the skill of **technical communication**: to listen and respond to another person in *real time*, to explain technical ideas understandably to others, to use your existing knowledge to fill in gaps in the presentation when possible, to ask for clarification when this is not. These skills will become increasingly important after you have graduated, regardless of the type of employment.

6. Find the right source for learning a topic and stick to it. This is a sort of corollary to the first point of advice about slowing down. If you want to learn a topic, shop around at first for the best book or other resource to learn it from. If you're taking a university course, the instructor has presumably done this already and chosen it as the course text. Once you've found this book, then slow down and work thoroughly. Every time you switch between books requires an investment of time and energy to learn the new author's terminology and read through the basics.

A good rule of thumb is often to choose the most advanced source that you can still understand completely. A book is too advanced if you have to accept many statements on faith, without your own ability to evaluate their correctness (with the help of the author's explanations, of course).

7. Math is hard; embrace it. When I was a student, I was no different than any of you. I recall sitting down to read my abstract algebra textbook and barely understanding a word of it. After reading through the relevant sections about ten times over the course of several weeks, I finally got the hang of it. It really is an incredible phenomenon how, with time and training, a topic goes from borderline impossible to "trivial".

But, once a topic gets too simple, it's time to move onto a bigger challenge. You might compare this to a video game whose difficulty adjusts to the skill of the player. From this point of view, math is guaranteed to be hard. Indeed, as you transition from student to researcher, you'll find that "being stuck" is the normal state of affairs. But simply recognizing this as something normal can be a great source of reassurance. It's okay to have to put in a big effort. If math wasn't difficult, if any random Joe could simply do it, then math as a discipline probably wouldn't exist, or at least wouldn't be worth doing.

8. But, if math isn't interesting, you're doing something wrong. There are many motivations for why one should learn mathematics. There are practical reasons related to real-world applications of math. You're probably interested in making a living using your skills with mathematics. However, I believe the *immediate* motivation for most people doing math is simply that's it's interesting. This means that you need to understand what you're learning and should focus on topics you find personally meaningful.

Tip 6 notwithstanding, math shouldn't be too hard too much of the time. Yes, it will take some work to learn new concepts. But if you're completely lost, then you're not gaining anything from the time you're investing. In such a case, don't be afraid to take a step back to something more familiar and go at a slower pace. In some cases, you might change to a different topic.

As you go throughout your mathematical education and career, don't lose contact with your own natural curiosity. This must be balanced, of course, with course requirements (as a student) and the expectation to produce original work in your chosen field (as a researcher). But I find I am most engaged in my work and most productive when I actively follow my own judgments about what is interesting.

9. Y**ou'll probably make a living someday by doing *one thing* better than (almost) anyone else.** This is perhaps more in the category of life advice than math advice, but I believe it is a good way to conclude this list. More accurately, "one thing" might be a small set of related skills. But the point is that, whatever you hope to do, you need to invest enough to become an expert in it. Bruce Lee expressed this idea more poetically: "I fear not the man who has practiced 10,000 kicks once, but I fear the man who had practiced one kick 10,000 times." Or, to paraphrase *Oceans 11*, "You only have one job!"

This piece of advice is a bit tricky: as an undergraduate student, you take a wide array of classes meant to give you a broad foundation for the rest of your life academically, professionally and personally. There is value in the breadth of your experience. For many of you, true specialization will come later in graduate school or in the workforce. So certainly one must find the right balance between depth and breadth.

Still, in the thick of your university studies, it's good to keep the end goal in mind. It's nice, say, to get an A on the final exam, but this is a means to an end rather than the end itself. Say you can get an A- with 50 hours of effort, but to improve this to an A requires 100 hours. Is the time worth it? If this class is part of your intended expertise, the answer is probably 'yes'. But for every class? Eventually you run into diminishing marginal returns: the extra effort just isn't worth the benefit.

More important than grades is to **become an expert**. This is more than just book knowledge. It includes deeper skills like communicating your knowledge, evaluating claims made by others, and applying your knowledge in creative directions. It includes finding your own individual, possibly original point of view on the subject. It includes the way you present yourself to others. Very importantly, if only from a practical point of view, you need to be recognized by other experts as such.

This is a process that takes time–probably many years. So the key is to be consistent and diligent. This might seem daunting. But there's also the flip side to this principle, which might be more reassuring: that you don't need to worry about being great at everything. This idea can help simplify our busy lives. As the saying goes, work smart, not hard. We are each one cog in the larger machine of society, and the main thing is for us to do our individual part well. If we do so, the machine does its part (ideally speaking; I recognize it's not always so clear-cut in practice) in multiplying our work and taking us further than we can on our own.

A university such as Stony Brook is one such larger machine. Take advantage of this structure. Participate in classes. Get to know your professors. Work with your professors if they'll let you (for the record, I'll always have an open door for an undergraduate looking for a research project). Be involved and be visible. You have four years as an undergraduate student to invest in yourself. Make your university education work for you. It's a big world out there, and we each have to find our place it in.

For those reading this who wish to become professional mathematicians, the application of this principle is clear. Your objective as you study isn't merely to "learn math", but to eventually master a given specialty. If you can say that you're better than anyone else on the planet at a certain type of math problem, you'll likely have a bright future as a mathematician. Of course, this specialization will mostly happen in graduate school. But there's no need for you to wait to get started. Why not get a head start on the competition? (Some disclaimers: as an undergraduate, it's healthy to have an idea of the broader field you'll go into and try your hand at a research project, but be flexible about your specific long-term specialty until you're well into graduate school. Make sure your focus is on a genuinely deep and interesting area of math; otherwise, your efforts may backfire. It helps to have the guidance of a professor or other mentor.)