Projection of $\mathbf{u}$ onto $\mathbf{v}$: $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

Identities: $\mathbf{u}\cdot\mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\|\,\cos(\theta)$ and $\|\mathbf{u}\times\mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\|\sin(\theta)$

Unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$, if $\mathbf{r}'(t) \neq \mathbf{0}$

Principal unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$, if $\mathbf{T}'(t) \neq \mathbf{0}$

Tangential component of acceleration: $a_{\mathbf{T}} = \dfrac{d}{dt}\|\mathbf{r}'(t)\| = \dfrac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{\|\mathbf{r}'(t)\|}$

Normal component of acceleration: $a_{\mathbf{N}} = \|\mathbf{r}'(t)\|\,\|\mathbf{T}'(t)\| = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|}$

Arc length: $s = \displaystyle\int_a^b \|\mathbf{r}'(t)\|\,dt$, Curvature: $K = \left\|\dfrac{d\mathbf{T}}{ds}\right\| = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

Second derivative test: $d = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{yx}(a,b) & f_{yy}(a,b) \end{vmatrix} = f_{xx}(a,b)f_{yy}(a,b) - (f_{xy}(a,b))^2.$

Cylindrical coordinates: $x = r\cos(\theta),\ y = r\sin(\theta),\ z = z.$

Spherical coordinates: $x = \rho\sin(\phi)\cos(\theta),\ y = \rho\sin(\phi)\sin(\theta),\ z = \rho\cos(\phi).$

Integration in cylindrical coordinates example:

$$\iiint_R f(x,y,z)\,dV = \int_a^b \int_{f_1(\theta)}^{f_2(\theta)} \int_{g_1(r,\theta)}^{g_2(r,\theta)} f(r\cos(\theta), r\sin(\theta), z) r\,dz\,dr\,d\theta.$$

Integration in spherical coordinates example:

$$\iiint_R f(x,y,z)\,dV = \int_a^b \int_{f_1(\theta)}^{f_2(\theta)} \int_{g_1(\phi,\theta)}^{g_2(\phi,\theta)} f(\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta.$$

Jacobian for the change of variables $x = g(u,v),\ y = h(u,v)$: $\left|\dfrac{\partial x}{\partial u}\dfrac{\partial y}{\partial v} - \dfrac{\partial x}{\partial v}\dfrac{\partial y}{\partial u}\right|$

Line integral of the vector field $\mathbf{F} = \langle M, N, P\rangle$ over C:

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C M\,dx + N\,dy + P\,dz = \int_C \mathbf{F}\cdot\mathbf{T}\,ds = \int_a^b \mathbf{F}(x(t),y(t),z(t))\cdot\mathbf{r}'(t)\,dt$$

Surface integral of $f(x,y,z)$ over S: $\displaystyle\iint_S f(x,y,z)\,dS = \iint_R f(x(u,v),y(u,v),z(u,v))\,\|\mathbf{r}_u\times\mathbf{r}_v\|\,dA.$

Flux of $\mathbf{F}$ across S: $\displaystyle\iint_S \mathbf{F}\cdot\mathbf{N}\,dS = \iint_D \mathbf{F}\cdot(\mathbf{r}_u\times\mathbf{r}_v)\,dA.$

$\text{curl }\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$, $\text{div }\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

Divergence theorem: $\displaystyle\iint_S \mathbf{F}\cdot\mathbf{N}\,dS = \iiint_Q \text{div }\mathbf{F}\,dV.$

Stokes's theorem: $\displaystyle\int_C \mathbf{F}\cdot\,d\mathbf{r} = \iint_S (\text{curl }\mathbf{F})\cdot\mathbf{N}\,dS.$