STONY BROOK UNIVERSITY

MAT200 Summer– Final
Logic, Language and Proof

Instructor: Lisa Marquand

06/08/2018

Name: ____
Student Number: ____________________________________

You have 3 hours to complete this exam. No mobile phones, calculators, smart watches, or notes of any kind are allowed.

This exam contains 14 pages (including this cover page) and 10 questions. Total of points is 100.

Good luck!

Distribution of Marks

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Show that $2^1 + 2^2 + \cdots + 2^n = 2^{n+1} - 2$ for all integers $n \geq 1$.

2. Consider the function $f : \mathbb{R} \times \mathbb{R}_{>0} \to \mathbb{R}$ defined by

$$f(x, y) = \frac{x}{y}.$$

   (a) (5 points) Prove or disprove that f is injective.

   (b) (5 points) Prove or disprove that f is surjective. If not, what is the image?

3. (10 points) You go out for a Fixed Menu Three course meal. There are 6 starter options, 3 main course options, and 5 options for dessert. How many possible meals (consisting of exactly one starter, one main, and one dessert) are there?

4. Give a proof or a counter example:
   (a) (3 points) $\forall r \in \mathbb{R}, \exists s \in \mathbb{R}$ such that $s^2 + 1 < r$.
   (b) (3 points) $\exists x \in \mathbb{R}$ such that $\forall r \in \mathbb{R}, (r \in \mathbb{Q} \Rightarrow rx \notin \mathbb{Q})$.
   (c) (4 points) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$ such that $x + y$ is even.

5. (10 points) The Fibonacci numbers are defined inductively by

$$F_0 = 0, F_1 = F_2 = 1, \text{ and } F_{n+1} = F_n + F_{n-1}, \forall n \in \mathbb{Z}, n \geq 1.$$

Prove by induction that

$$F_{n+1}^2 - F_n F_{n+2} = (-1)^n,$$

for all integers $n \geq 1$.

6. (10 points) There are 75 students who are enrolled in analysis, algebra, or topology. Each class has exactly 40 students. 12 are taking only analysis, 13 are taking only algebra, and 14 are taking only topology. How many students are taking all three?

   Hint: Recall that for three sets X, Y, Z, the inclusion-exclusion principle says

$$|X \cup Y \cup Z| = |X| + |Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z|.$$

7. (10 points) Prove that from any subset of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ of cardinality 6 has two numbers which sum to 11.

8. (10 points) Let $n \in \mathbb{Z}$. Prove that n is odd if and only if there exists $k \in \mathbb{Z}$ such that $n = 2k+1$.
   Hint: You can assume that for a finite set $A \subset \mathbb{Z}$, then A has a maximum.

9. (10 points) For $(a, b), (c, d) \in \mathbb{R}^2$, define $(a, b) \sim (c, d)$ if $a^2 + b^2 = c^2 + d^2$.

   Prove that $\sim$ is an equivalence relation.

   List the elements in the equivalence class of $(0, 0)$.

10. (10 points) Find all integers x for which $5x \equiv 12 \mod 19$.

    Hint: Apply Euclidean Algorithm.

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