EXAM 4 (FALL 2019)

1. (1) $\dfrac{\partial F}{\partial x} = \dfrac{(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = \boxed{\dfrac{-x^2+y^2}{(x^2+y^2)^2}}$

$\dfrac{\partial F}{\partial y} = \dfrac{-x(2y)}{(x^2+y^2)^2} = \boxed{\dfrac{-2xy}{(x^2+y^2)^2}}$

(2) $F(x,y,z) = \ln(x+2z) + 3xy^2 - 200 = 0$

$\dfrac{\partial F}{\partial x} = \dfrac{1+2\frac{\partial z}{\partial x}}{x+2z} + 3y^2 = 0 \qquad \dfrac{\partial F}{\partial y} = 6xy + \dfrac{2\frac{\partial z}{\partial y}}{x+2z} = 0$

$\dfrac{\partial z}{\partial x} = \boxed{\frac{1}{2}\left(-3y^2(x+2z) - 1\right)} \qquad \dfrac{\partial z}{\partial y} = \dfrac{-6xy(x+2z)}{2}$

$= \boxed{-3xy(x+2z)}$

2. (1) $\nabla F = \langle F_x, F_y, F_z \rangle = \left\langle \dfrac{1}{x+y^2+z^3}, \dfrac{2y}{x+y^2+z^3}, \dfrac{3z^2}{x+y^2+z^3} \right\rangle$

(2) $\nabla F = \langle F_x, F_y \rangle = \langle \cos(xy)y + e^{xy}y, \cos(xy)x + e^{xy}x \rangle$

$\nabla F(0,0) = \langle 1\cdot 0 + 1\cdot 0, 1\cdot 0 + 1\cdot 0 \rangle = \langle 0,0 \rangle$

(3) $G(x,y,z) = xy + yz + xz - 9 = 0$

$\nabla G = \langle y+z, x+z, x+y \rangle$

$\nabla G(1,1,4) = \langle 5,5,2 \rangle$

Tangent plane: $5(x-1) + 5(y-1) + 2(z-4) = 0$

$\boxed{5x + 5y + 2z - 18 = 0}$

Normal Line: $\boxed{x = 1+5t,\ y = 1+5t,\ z = 4+2t}$