MIDTERM 3 (FALL 2018)

1. (a) $\vec{v} = \dfrac{\langle 2, 1, -2\rangle}{\sqrt{4+1+4}} = \langle 2/3, 1/3, -2/3\rangle$

(b) $2(x+1) + 1(y-2) - 2z = 0$

$2x + y - 2z = 0$

(c) $\dfrac{\|\langle 2,1,-2\rangle \times \langle 4,1,3\rangle\|}{\|\langle 2,1,-2\rangle\|} = \dfrac{15}{3} = 5$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -2 \\ 4 & 1 & 3 \end{vmatrix} = (3+2)\vec{i} - (6+8)\vec{j} + (2-4)\vec{k} = 5\vec{i} - 14\vec{j} - 2\vec{k}$$

$\sqrt{25+196+4} = \sqrt{225} = 15$

4. $\dfrac{\partial f}{\partial y} = z^2 \cos\left(\dfrac{xy}{z^2}\right)\dfrac{x}{z^2} = x\cos\left(\dfrac{xy}{z^2}\right)$

$\dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial z}\right) = \dfrac{\partial}{\partial z}\left(\dfrac{\partial f}{\partial y}\right) = -x\sin\left(\dfrac{xy}{z^2}\right)\left(\dfrac{-2xy}{z^3}\right)$

$= \dfrac{2x^2y}{z^3}\sin\left(\dfrac{xy}{z^2}\right)$

$\dfrac{\partial f}{\partial x} = y\cos\left(\dfrac{xy}{z^2}\right)$

$\dfrac{\partial^2 f}{\partial x^2} = -\dfrac{y^2}{z^2}\sin\left(\dfrac{xy}{z^2}\right)$

2 (a) $\vec{r}\,'(t) = \langle 4\cos(t^2-\pi t)(2t-\pi),\ 8e^{4(t-\pi)}\rangle$

$\vec{r}\,'(\pi) = \langle 4\cos(0)(\pi),\ 8e^0\rangle = \langle 4\pi, 8\rangle$ $\quad \|\vec{r}\,'(\pi)\| = \sqrt{16\pi^2+64} = 4\sqrt{\pi^2+4}$

$T(\pi) = \left\langle \dfrac{\pi}{\sqrt{\pi^2+4}}, \dfrac{2}{\sqrt{\pi^2+4}}\right\rangle$

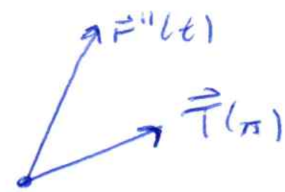
(b) $N(\pi)$ is a unit vector normal to $T(\pi)$, so either

$\left\langle \dfrac{-2}{\sqrt{\pi^2+4}}, \dfrac{\pi}{\sqrt{\pi^2+4}}\right\rangle$ or $\left\langle \dfrac{2}{\sqrt{\pi^2+4}}, \dfrac{-\pi}{\sqrt{\pi^2+4}}\right\rangle$.

Now $\vec{r}\,''(t) = \langle -4\sin(t^2-\pi t)(2t-\pi)^2 + 8\cos(t^2-\pi t),\ 32e^{4(t-\pi)}\rangle$

$\vec{r}\,''(\pi) = \langle 0 + 8, 32\rangle = \langle 8, 32\rangle$

Observe that $\vec{r}\,''(t)$ points <u>left</u> from $\vec{T}(t)$.



So we must have $N(\pi) = \left\langle \dfrac{-2}{\sqrt{\pi^2+4}}, \dfrac{\pi}{\sqrt{\pi^2+4}}\right\rangle$

3 $\vec{R}'(t) = \langle 2e^t, -2e^{-t}, 2\sqrt{2}\rangle$

$\|\vec{R}'(t)\| = \sqrt{4e^{2t} + 4e^{-2t} + 8} = 2\sqrt{(e^t+e^{-t})^2} = 2(e^t+e^{-t})$

$\displaystyle\int_0^3 \|\vec{R}'(t)\|\,dt = \int_0^3 2(e^t+e^{-t})\,dt = 2(e^t - e^{-t})\Big|_0^3 = 2(e^3-e^{-3}) - 2(1-1)$

$= 2(e^3 - e^{-3})$