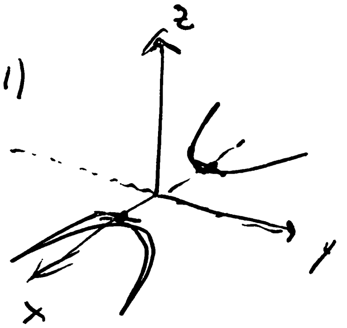
EXAM 2 (FALL 2017)

1. (1) $\cos(\theta) = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \dfrac{0-2-1}{\sqrt{2}\,\sqrt{6}} = \boxed{\dfrac{-3}{\sqrt{12}}} = \boxed{\dfrac{-\sqrt{3}}{2}}$

(2) $\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = (1-2)\vec{i} - (0-1)\vec{j} + (0+1)\vec{k} = -\vec{i}+\vec{j}+\vec{k} = \langle -1, 1, 1\rangle$

$\vec{u}\cdot(\vec{u}\times\vec{v}) = \boxed{0}$.

2. (1)



Take $x=r$. Then $r^2 - 2y^2 = 1$

$\Longrightarrow\ x^2 + z^2 - 2y^2 = 1$

(cylindrical coordinates about the y-axis)

Note: We didn't emphasize surfaces of rotation, so there won't be a similar problem on our test.

(2) $\boxed{4^2 = (x+1)^2 + (y-1)^2 + (z-1)^2}$

3. (1) $\vec{n} = \langle -1, 1, 1\rangle$ $\qquad -(x-0) + (y-1) + (z-2) = 0$

$-x + y + z = 3$

(2) Obrerve that $Q(0,0,0)$ belongs to the plane.

$\vec{n} = \langle 2, 3, 1\rangle \qquad \dfrac{\vec{n}}{\|\vec{n}\|} = \dfrac{\vec{n}}{\sqrt{14}} = \left\langle \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}, \dfrac{1}{\sqrt{14}} \right\rangle$

Project $\overrightarrow{QP}$ onto $\vec{n}$: $\quad \langle 1, -1, 2\rangle \cdot \dfrac{\vec{n}}{\|\vec{n}\|} = \dfrac{2}{\sqrt{14}} - \dfrac{3}{\sqrt{14}} + \dfrac{2}{\sqrt{14}} = \boxed{\dfrac{1}{\sqrt{14}}}$

4. (1) $\vec{r}\,'(t) = \langle -3\sin t, 3\cos t, 2t\rangle$

$\vec{a}(t) = \vec{r}\,''(t) = \langle -3\cos t, -3\sin t, 2\rangle$

(2) $\vec{r}\,'(1) = \langle -3\sin(1), 3\cos(1), 2\rangle \qquad \|\vec{r}\,'(1)\| = \sqrt{9+4} = \sqrt{13}$

$\vec{T}(1) = \left\langle \dfrac{-3}{\sqrt{13}}\sin(1), \dfrac{3}{\sqrt{13}}\cos(1), \dfrac{2}{\sqrt{13}} \right\rangle$

$a_T = \vec{a}(1)\cdot\vec{T}(1) = \dfrac{9}{\sqrt{13}}\sin(1)\cos(1) - \dfrac{9}{\sqrt{13}}\cos(1)\sin(1) + \dfrac{4}{\sqrt{13}} = \boxed{\dfrac{4}{\sqrt{13}}}$

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5. (1) $\vec{r}\,'(t) = \langle 3\cos(t), 4, 3\sin(t)\rangle$

$\vec{r}\,'(0) = \langle 3, 4, 0\rangle$

$x = 3t, \quad y = 1 + 4t, \quad z = -3$

(2) $$\int_{-\pi}^{\pi} \|\vec{r}\,'(t)\|\,dt = \int_{-\pi}^{\pi} \sqrt{9+16}\,dt = 2\pi\sqrt{25} = 10\pi$$

6. (1) $\vec{r}\,'(t) = \langle 2\sin(t), -2\cos(t), 0\rangle + \vec{v}_0$

$\vec{r}\,'(0) = \langle 0, -2, 0\rangle + \vec{v}_0 = \langle 0, 1, 1\rangle$

$\Longrightarrow \vec{v}_0 = \langle 0, 3, 1\rangle$

$\vec{r}\,'(t) = \langle 2\sin(t), -2\cos(t) + 3, 1\rangle$

$\vec{r}(t) = \langle -2\cos(t), -2\sin(t) + 3t, t\rangle + \vec{r}_0$

$\vec{r}(0) = \langle -2, 0, 0\rangle + \vec{r}_0 = \langle -1, 0, 0\rangle$

$\Longrightarrow \vec{r}_0 = \langle 1, 0, 0\rangle$

$\vec{r}(t) = \langle -2\cos(t) + 1, -2\sin(t) + 3t, t\rangle$

(2) $\vec{r}\,'(\pi) = \langle 0, 5, 1\rangle \qquad T(\pi) = \dfrac{\vec{r}\,'(\pi)}{\|\vec{r}\,'(\pi)\|} = \left\langle 0, \dfrac{5}{\sqrt{26}}, \dfrac{1}{\sqrt{26}}\right\rangle$