HANDOUT

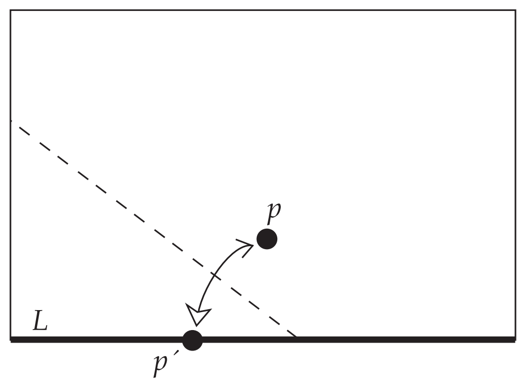
Exploring a Basic Origami Move

Origami books display many different folding moves that can be made with paper. One common move, especially in geometric folding, is the following:

> Given two points p_1 and p_2 and a line L, fold p_1 onto L so that the resulting crease line passes through p_2.

Let's explore this basic origami operation by seeing exactly what is happening when we fold a point to a line.

Activity: Take a sheet of regular writing paper, and let one side of it be the line L. Choose a point p somewhere on the paper, perhaps like below. Your task is to fold p onto L over and over again.



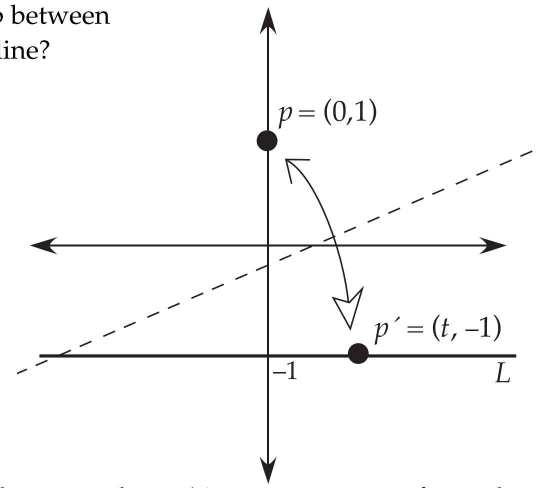
It is easier, actually, to fold L to p, by bending the paper until L touches p and then flattening the crease. Do this many times—as many as you can stand!—choosing different points p' where p lands on L.

Question 1: Describe, as clearly as you can, exactly what you see happening. What are the crease lines forming? How does your choice of the point p and the line L fit into this? Prove it.

Now we'll try to find the equation for the curve you discovered.

First, let's define where things lie on the xy-plane. Let the point $p = (0, 1)$ and let L be the line $y = -1$. Now suppose that we fold p to a point $p' = (t, -1)$ on the line L, where t can be any number.

Question 2: What is the relationship between the line segment $\overline{pp'}$ and the crease line? What is the slope of the crease line?



Question 3: Find an equation for the crease line. (Write it in terms of x and y, although it will have the t variable in it as well.)

Question 4: Your answer to Question 3 should give you a **parameterized family** of lines. That is, for each value of t that you plug in, you'll get a different crease line. For a fixed value of t, find the point on the crease line that is **tangent** to your curve from Question 1.

Question 5: Now find the equation for the curve from Question 1.

Question 6: What happens if we use a circle instead of a line?