5. The *Fibonacci sequence* is a sequence of positive integers defined by recurrence as follows: $F_0 = 1$, $F_1 = 1$ and for each integer i larger than one,

$$F_i = F_{i-1} + F_{i-2}.$$

Use `Maple` to find the first 100 terms of the sequence **without computing these terms one by one** (Hint: See `Fibonacci`). Use the sequence you found to compute a new sequence F_i/F_{i-1} and try to guess what is the limit of that new sequence. EXTRA CREDIT: Prove the sequence F_i/F_{i-1} converges to the limit you guessed (You may need to refine your guess to do so).

6. Fit the points $(-1.9, -4.7), (-0.8, 1.2), (0.1, 2.8), (1.4, -1.2), (1.8, -3.5)$ by means of a quadratic function $f(x) = ax^2 + bx + c$, using the least square method. First, do this step by step, as we did in class; then, use the built-in `Maple` command, described in the notes. Check that the two solutions agree.

7. Fit the set of points

$$(1.02, -4.30), (1.00, -2.12), (0.99, 0.52), (1.03, 2.51), (1.00, 3.34), (1.02, 5.30)$$

with a line, using the least square method we used in class. You will see that this is not a good fit. Think of a better way to find a line which *is* a good fit and use `Maple` to do it. Explain in your solution why you think your better way is indeed better.

8. The set of eight points

$$\begin{array}{cccc} (2.073, 5.794) & (-1.931, 1.316) & (.3959, 3.441) & (3.353, 8.950) \\ (4.267, 12.65) & (3.167, 8.669) & (2.876, 6.992) & (-1.245, 1.616) \end{array}$$

approximate an exponential function of the form $f(x) = ae^{bx}$. Use least squares to find good values for a and b. Plot the data points and your curve on the same axes.

If you don't want to retype the points, you can load them from the file `expdata.txt` from the problems area on the class web page.

NOTE: Neither of the next two problems intrinsically involve `Maple`, except as a word processor to write your solution (although you can use it to help with calculations if you want). If you prefer, you are welcome to turn in a printed or handwritten version instead, though a worksheet is slightly preferred.

9. Following Section 6 (Fitting a Circle) of the notes, prove that if we describe the circle of center (a, b) and radius r using the parameters (a, b, k), with $k = a^2 + b^2 - r^2$, rather than the more natural parameters (a, b, r), then the error function $H(a, b, k) = E(a, b, \sqrt{a^2 + b^2 - k})$ is quadratic in a, b and k. What does this imply about the number of critical points?

10. (EXTRA CREDIT) With reference to the previous problem, show that for $r > 0$, the transformation $(a, b, r) \mapsto (a, b, k)$ is a valid change of variables, that is, it is a diffeomorphism (a one-to-one function with continuous nonzero derivatives). This should help you prove that $E(a, b, r)$ has only one "physical" critical point, which is a minimum, and is mapped, through the transformation, into the unique critical point of $H(a, b, k)$.

11. The set of twelve points

$$\begin{array}{cccc}
(-2.256, 0.879) & (-1.764, 5.800) & (-0.684, -0.854) & (-0.776, 6.750) \\
(3.718, 7.394) & (0.081, -1.315) & (-2.357, 4.534) & (6.485, 4.021) \\
(6.518, 3.999) & (2.818, 7.689) & (1.788, -1.668) & (-2.720, 2.719)
\end{array}$$

approximate a circle with an unknown radius and center at $(2, 3)$. What is the "best" value for r corresponding to this data? Explain your answer. Plot the resulting circle and the data points on the same graph.

(Note that if you fit the data using the method described in section 6 of the notes, you'll get a somewhat different radius. Using an unknown center gives one at approximately $(1.970, 3.002)$ due to the noise in the data.)

If you don't want to retype the points, you can load them from the file `circdata.txt` from the problems area on the class web page.