86.31 Proof without Words: <tex-math>$\sum_{r=1}^{n}r^{3}=\left(\sum_{r=1}^{n}r\right)^{2}$</tex-math>
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operation of removing the last object of each sequence and putting it at the head. Then f^{p^n} becomes again the identity map on X and the sequences satisfying $x = f^{p^{n-1}}(x)$ are those which repeat p times arbitrary subsequences of length p^{n-1} so that there are $k^{p^{n-1}}$ such sequences. Thus we get a general version of *Fermat's little theorem* ([2, p. 66]); $|X| = k^{p^n} \equiv k^{p^{n-1}} \pmod{p^n}$ or, if k is relatively prime to p, $k^{p^{n-1}(p-1)} \equiv 1 \pmod{p^n}$.

References

1. G. E. Andrews, *Number theory*, Saunders (1971) [Dover (1994)].
2. G. H. Hardy and E. M. Wright, *An introduction to the theory of numbers*, Oxford University Press (1979).

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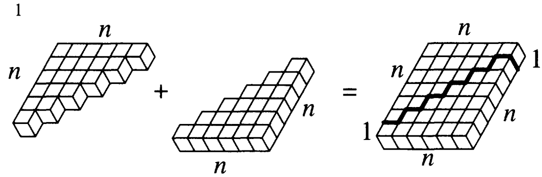
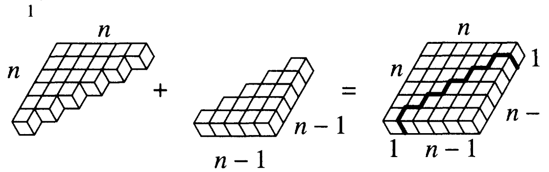
It is well known that the formula for the sum of cubes of the first n natural numbers $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2 = \left(\sum_{r=1}^{n} r\right)^2$ so maybe it is possible to show this geometrically. Here is an outline illustration based on putting cubes together.

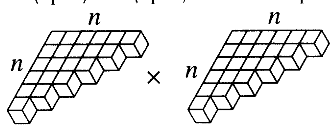
1. $\sum_{1}^{n} r + \sum_{1}^{n} r = n(n+1)$.



2. $\sum_{1}^{n} r + \sum_{1}^{n-1} r = n^2$.



3. $$\left(\sum_1^n r\right)^2 = \left(\sum_1^n r\right) \times \left(\sum_1^n r\right) = n \times \sum_1^n r + \left(\sum_1^{n-1} r\right) \times \left(\sum_1^n r\right)$$



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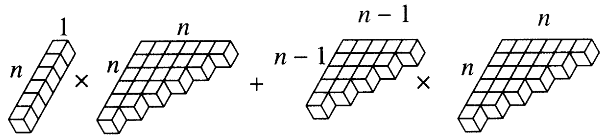

$$= n \times \sum_1^n r + n \times \sum_1^{n-1} r + \left(\sum_1^{n-1} r\right) \times \left(\sum_1^{n-1} r\right)$$

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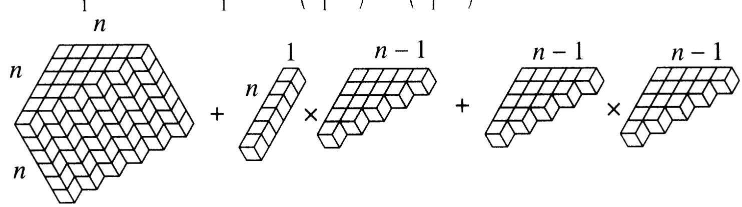

$$= n^3 + \left(\sum_1^{n-1} r\right)^2.$$

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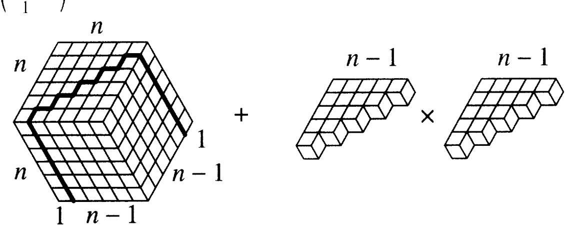

$$= n^3 + \left(\sum_1^{n-1} r\right)^2.$$

4. Similarly $\left(\sum_1^{n-1} r\right)^2 = (n - 1)^3 + \left(\sum_1^{n-2} r^2\right)$ etc. to get

$$\left(\sum_1^n r\right)^2 = n^3 + (n - 1)^3 + \dots + 2^3 + 1^3 = \sum_1^n r^3.$$

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