

MATHEMATICAL ASSOCIATION



supporting mathematics in education

---

86.31 Proof without Words:  $\sum_{r=1}^n r^3 = \left(\sum_{r=1}^n r\right)^2$

Author(s): Peter Holmes

Source: *The Mathematical Gazette*, Vol. 86, No. 506 (Jul., 2002), pp. 267-268

Published by: The Mathematical Association

Stable URL: <http://www.jstor.org/stable/3621854>

Accessed: 24/03/2010 21:15

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=mathas>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



*The Mathematical Association* is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

operation of removing the last object of each sequence and putting it at the head. Then  $f^{p^n}$  becomes again the identity map on  $X$  and the sequences satisfying  $x = f^{p^{n-1}}(x)$  are those which repeat  $p$  times arbitrary subsequences of length  $p^{n-1}$  so that there are  $k^{p^{n-1}}$  such sequences. Thus we get a general version of *Fermat's little theorem* ([2, p. 66]);  $|X| = k^{p^n} \equiv k^{p^{n-1}} \pmod{p^n}$  or, if  $k$  is relatively prime to  $p$ ,  $k^{p^{n-1}(p-1)} \equiv 1 \pmod{p^n}$ .

*References*

1. G. E. Andrews, *Number theory*, Saunders (1971) [Dover (1994)].
2. G. H. Hardy and E. M. Wright, *An introduction to the theory of numbers*, Oxford University Press (1979).

SHINJI TANIMOTO

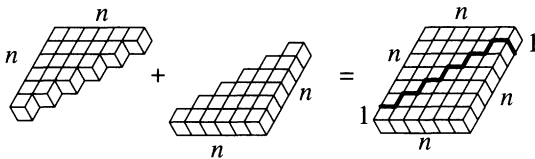
*Dept of Mathematics, Kochi Joshi University, Kochi 780-8515, Japan*

*e-mail: tanimoto@cc.kochi-wu.ac.jp*

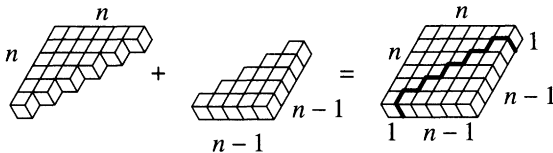
**86.31 Proof without words:**  $\sum_{r=1}^n r^3 = \left(\sum_{r=1}^n r\right)^2$

It is well known that the formula for the sum of cubes of the first  $n$  natural numbers  $\sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2 = \left(\sum_{r=1}^n r\right)^2$  so maybe it is possible to show this geometrically. Here is an outline illustration based on putting cubes together.

$$1. \quad \sum_1^n r + \sum_1^n r = n(n+1).$$



$$2. \quad \sum_1^n r + \sum_1^{n-1} r = n^2.$$



$$\begin{aligned}
 3. \quad \left(\sum_1^n r\right)^2 &= \left(\sum_1^n r\right) \times \left(\sum_1^n r\right) = n \times \sum_1^n r + \left(\sum_1^{n-1} r\right) \times \left(\sum_1^n r\right) \\
 &= \begin{array}{c} n \\ \text{---} \\ n \end{array} \times \begin{array}{c} n \\ \text{---} \\ n \end{array} + \begin{array}{c} n-1 \\ \text{---} \\ n-1 \end{array} \times \begin{array}{c} n \\ \text{---} \\ n \end{array} \\
 &= n \times \sum_1^n r + n \times \sum_1^{n-1} r + \left(\sum_1^{n-1} r\right) \times \left(\sum_1^{n-1} r\right) \\
 &= \begin{array}{c} n \\ \text{---} \\ n \end{array} \times \begin{array}{c} n \\ \text{---} \\ n \end{array} + \begin{array}{c} 1 \\ \text{---} \\ 1 \end{array} \times \begin{array}{c} n-1 \\ \text{---} \\ n-1 \end{array} + \begin{array}{c} n-1 \\ \text{---} \\ n-1 \end{array} \times \begin{array}{c} n-1 \\ \text{---} \\ n-1 \end{array} \\
 &= n^3 + \left(\sum_1^{n-1} r\right)^2. \\
 &= \begin{array}{c} n \\ \text{---} \\ n \end{array} \times \begin{array}{c} n \\ \text{---} \\ n \end{array} + \begin{array}{c} n-1 \\ \text{---} \\ n-1 \end{array} \times \begin{array}{c} n-1 \\ \text{---} \\ n-1 \end{array} \\
 &= n^3 + \left(\sum_1^{n-1} r\right)^2.
 \end{aligned}$$

4. Similarly  $\left(\sum_1^{n-1} r\right)^2 = (n-1)^3 + \left(\sum_1^{n-2} r^2\right)$  etc. to get

$$\left(\sum_1^n r\right)^2 = n^3 + (n-1)^3 + \dots + 2^3 + 1^3 = \sum_1^n r^3.$$

PETER HOLMES

*Royal Statistical Society Centre for Statistical Education,  
Nottingham-Trent University, Burton Street, Nottingham NG1 4BU*