In this note I observed that although this theorem was known to Hadamard (1906), it does not appear to be "well-known" at the present time, and soon after the note went to press I learned that the theorem was rediscovered by R. S. Palais in the course of developing propositions of a more general character. See [**2**, p. 128–129].

Professor Palais also suggests the following simplifications to the proof: In my note I appealed to the standard results of degree theory to establish that f is onto; but this is not necessary, for one can easily show that a proper map between manifolds sends closed sets into closed sets. (For a more general statement of this fact see [**3**].) But the non-vanishing of the Jacobian implies that f is a local homeomorphism and therefore sends open sets into open sets. Hence $f(M_1)$ is both an open and closed (nonempty) subset of M_2; i.e., $f(M_1) = M_2$.

References

1. W. B. Gordon, On the diffeomorphisms of euclidean space, this MONTHLY, 79 (1972) 755–759.

2. R. S. Palais, Natural operations on differential forms, Trans. Amer. Math. Soc., 92 (1959) 125–141.

3. ———, When proper maps are closed, Proc. Amer. Math. Soc., 24 (1970) 835–836.

RESEARCH PROBLEMS

EDITED BY RICHARD GUY

In this Department the Monthly presents easily stated research problems dealing with notions ordinarily encountered in undergraduate mathematics. Each problem should be accompanied by relevant references (if any are known to the author) and by a brief description of known partial results. Manuscripts should be sent to Richard Guy, Department of Mathematics, Statistics, and Computing Science, The University of Calgary, Calgary 44, Alberta, Canada.

HOW UNEXPECTED IS THE PRIME NUMBER THEOREM?

M. D. HIRSCHHORN, University of New South Wales, Australia

Let $p_1, \cdots, p_n$ be the first n primes. Let x be chosen randomly from among the integers greater than p_n. The probability that x is divisible by p_i is $1/p_i$, so the probability that x is divisible by none of $p_1, \cdots, p_n$ is

$$\left(1 - \frac{1}{p_1}\right) \cdots \left(1 - \frac{1}{p_n}\right).$$

If we make the unjustified assumption that the property of being divisible by none of $p_1, \ldots, p_n$ is held randomly by numbers greater than p_n with probability $(1 - 1/p_1) \cdots (1 - 1/p_n)$, then the probability that

$$p_{n+1} - p_n = r$$

is

$$\left(1 - \left(1 - \frac{1}{p_1}\right) \cdots \left(1 - \frac{1}{p_n}\right)\right)^{r-1} \left(1 - \frac{1}{p_1}\right) \cdots \left(1 - \frac{1}{p_n}\right).$$

So the expected value of $p_{n+1} - p_n$ is

$$\sum_{r=1}^{\infty} r\left(1 - \left(1 - \frac{1}{p_1}\right) \cdots \left(1 - \frac{1}{p_n}\right)\right)^{r-1} \left(1 - \frac{1}{p_1}\right) \cdots \left(1 - \frac{1}{p_n}\right)$$

$$= 1 \bigg/ \left(1 - \frac{1}{p_1}\right) \cdots \left(1 - \frac{1}{p_n}\right).$$

Accordingly, we define the series of **problimes** by

$$q_1 = 2, \; q_{n+1} = q_n + 1 \bigg/ \left(1 - \frac{1}{q_1}\right) \cdots \left(1 - \frac{1}{q_n}\right).$$

The interesting question that arises is, what is the asymptotic behavior of the q_n? In particular, is it true that

$$q_n \sim n \log_e n?$$

If this *is* true, then perhaps the prime number theorem is a little less surprising, since it is a consequence of the prime number theorem that

$$p_n \sim n \log_e n.$$

I have been able to prove that

(1) q_n/n is unbounded,

(2) $q_n/n^{1+\varepsilon} \to 0$ as $n \to \infty$ for any fixed $\varepsilon > 0$,

but I have not been able to prove that q_n/n is eventually monotonic increasing.

It is clear that the problimes soon become non-integral. It may be more attractive to study the various integral sequences defined by

$$q_1 = 2, \, q_{n+1} = q_n + F\left(1 \bigg/ \left(1 - \frac{1}{q_1}\right) \cdots \left(1 - \frac{1}{q_n}\right)\right),$$

where

$F(x) = [x]$, the greatest integer not greater than x,
$F(x) = \langle x \rangle$, the closest integer to x, or
$F(x) = \{x\}$, the least integer not less than x.

The first few terms of each of these sequences are given in the following table, together with the primes for comparison:

n	1	2	3	4	5	6	7	8	9	10	11
p_n	2	3	5	7	11	13	17	19	23	29	31
q_n^*	2.0	4.0	6.7	9.8	13.3	17.1	21.1	25.3	29.7	34.2	38.9
$q_n,[]$	2	4	6	9	12	15	19	23	27	31	35
$q_n,\langle\rangle$	2	4	7	10	13	17	21	25	29	34	39
$q_n,\{\}$	2	4	7	11	15	19	23	28	33	38	43

n	12	13	14	15	16	17	18	19	20	21
p_n	37	41	43	47	53	59	61	67	71	73
q_n	43.7	48.6	53.6	58.7	63.9	69.2	74.6	80.0	85.5	91.1
$q_n,[]$	40	45	50	55	60	65	70	75	80	86
$q_n,\langle\rangle$	44	49	54	59	64	69	74	79	84	90
$q_n,\{\}$	48	53	58	63	68	73	79	85	91	97

* to 1 decimal place

P. Erdös (written communication) believes that he can prove by Tauberian arguments that $(q_{n+1} - q_n) / \log_e n \to 1$, and hence that $q_n/n \log_e n \to 1$.

I am indebted to the referee and to Richard K. Guy for their helpful comments and suggestions.

CLASSROOM NOTES

EDITED BY ROBERT GILMER

Material for this Department should be sent to David Roselle, Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803.

THE INDECOMPOSABILITY OF THE DYADIC SOLENOID

S. B. NADLER, JR., University of North Carolina at Charlotte

In a beginning topology course students are sometimes confronted with the notion of "indecomposability" for continua (a continuum is *indecomposable* [4, p. 139] provided it cannot be written as the union of two proper subcontinua). There are proofs in the literature (see, for example, [3], [5], and material related