Prime Number and Cosmical Number
Author(s): Robert S. Hartman
Source: *Philosophy of Science*, Vol. 9, No. 2 (Apr., 1942), pp. 190-196
Published by: The University of Chicago Press on behalf of the Philosophy of Science Association
Stable URL: http://www.jstor.org/stable/184431
Accessed: 24/03/2010 17:46

PRIME NUMBER AND COSMICAL NUMBER

ROBERT S. HARTMAN

The conformity of mathematics and physics has so far been taken for granted. Philosophical explanations of that fundamental fact have never been satisfactory, mathematical explanations never had been attempted. In the following a fundamental theorem for the conformity of mathematics and physics will be demonstrated.

Mathematics can be defined as the science of Number, physics as the science of Matter. The elementary constituents of mathematics are the prime numbers, those of matter the particles, particularly protons and electrons. The only essential property of these elements as such is their existence, given within a given complex or space. Their fundamental relation is their distribution within their respective spaces. The theorem of the distribution of primes in the arithmetical series of numbers is the fundamental theorem of mathematics. The fundamental theorem of physics would be a theorem of the distribution of particles in geometrical space. If it were shown that both theorems were equal, the essential conformity of mathematics and physics would be mathematically demonstrated.

The distribution of prime numbers is given by the famous Prime Number Theorem, described by Abel in 1823 as "the most remarkable in the whole mathematics." In the form given to it by Tchebychef in 1852 it runs:

$$P_x = a \frac{x}{\log x}, \tag{I}$$

where P_x is the number of primes $\leqq x$, and a between 0.92129 and 1.10555. The logarithm is the natural logarithm. For the practical use of the theorem we have to choose a certain value for a. For reasons which will become apparent later, we choose the value 0.96489. Our formula for the distribution of primes then is

$$P_x = 0.96489 \frac{x}{\log x}. \tag{II}$$

As is known, the relativity theory establishes a relation between the geometrical properties of space and the *mass* of the universe. From the latter the number of particles can be derived subsequently, either by the comparatively simple method of dividing the mass of the world by that of the proton, or the more elaborate method of Eddington. Eddington, combining quantum and relativity theory, not only supplies us with the number N of particles in the universe, the COSMICAL NUMBER, but also with the two other pure number constants of nature, the mass-ratio m_p/m_e, between 1846 and 1847, where m_p is the mass of the proton and m_e that of the electron, and the reciprocal of the fine-structure constant, $hc/2\pi e^2$, which is about 137, the most remarkable number in physical science. In this formula h is Planck's constant, c the velocity of light, e the electronic charge. There exists no derivation of the cosmical number *directly* from space.

Eddington's value for N is $1.5747 \cdot 10^{79}$. The entire 80 digits are given in *the Philosophy of Physical Science.*[1] The number is slightly different from the one given in *The Relativity Theory of Protons and Electrons*,[2] which is $1.5727 \cdot 10^{79}$. While in the latter case he calculates the number as $135.82 \cdot 2^{256}$, he deduces it in the first case as $136 \cdot 2^{256}$. N is the number of all the electrons *or* protons in the universe and corresponds to a volume $V = R^3\pi^2$, where R is the initial radius of the universe (the number of all particles being 2 N and the corresponding volume $V = 2R^3\pi^2$). According to Eddington's equation $\frac{1}{2}\pi R = kM/c^2 = \frac{1}{2} kN (m_p + m_e)/c^2$, where k is the Einstein constant of gravitation, to the above value N belongs the value $R = 1.234 \cdot 10^{27}$ cm. Hence, according to Friedman-Lemaître-Hubble's equation $R = c/\sqrt{3\alpha}$, where α is the Hubble constant for the velocity of nebular recession per megaparsec, $\alpha = v/3.1 \cdot 10^{24}$ km/sec/mp, v is found to be 432 km/sec/mp.

This value for the speed of recession is obviously too small. The astronomical data for the recession of nebulae are derived from the displacement of the spectral lines toward the red end of the nebular spectrum. This "red-shift," independent of the wave length and with all the characteristics of a Doppler effect, "is a very well-marked and easily observable phenomenon."[3] "With the 100-inch telescope and a spectrograph of small dispersion (418 A/mm at $\lambda = 4500$), it is possible to photograph the spectra of nebulae as faint as the 18th apparent magnitude. Four lines H, K, Hδ . . . and the G band . . . are usually clearly shown and determinations for over 170 nebulae are now available": The Doppler velocity v is given by $v = c\delta$ where $\delta = d\frac{\lambda}{\lambda}$, denoting the displacement in wave-length λ; c is the velocity of light. Plotting $\log_{10} v$ against the apparent magnitude m of the nebulae observed, Hubble and Humason[4] up to 1934 gave the relation between these quantities as $\log_{10} v = 0.2m + 0.507 \pm 0.012$. . . . In 1937 the formula has been revised owing to the accumulation of some further data and stands $\log_{10} v = 0.2m_c + 0.77 \ldots$, where m_c is the apparent magnitude corrected for red shift.[5] McVittie, regarding the constant terms 0.51 and 0.77 as minimum and maximum values and using the uncorrected magnitude m in both cases, deduces from the two formulas a minimum velocity of recession $v = 465$ km/sec/mp and a maximum velocity $v = 851$ km/sec/mp. More exact and in conformity with the observational material it would be to regard the second formula as definite. Considering the corrected magnitude m_c, which is slightly less than m and therefore decreases somewhat the constant

[1] New York: The Macmillan Company, 1939, page 170.

[2] Cambridge: At the University Press, 1936, page 272.

[3] McVittie, *Cosmological Theory*, London: Methuen and Co., Ltd., 1937, page 9. There also following quotations.

[4] *Astrophys. J. 74*, 43, 1931. *Proc. Nat. Acad. Sci. Philad. 15*, 168, 1929; *20*, 264, 1934.

[5] *Astrophys. J. 84*, 158 and 270, 1936.

factor 0.77, the definite speed of recession would have to be determined as v $\sim$ 800 km/sec/mp.

We assert that the distribution of prime numbers in an arithmetical complex is equal to the distribution of particles in the geometrical complex called space. The Prime Number Theorem we put as

$$P_x = 0.96489 \frac{x}{\log x}. \qquad \text{I}$$

Regarding the number of primes P_x within the complex x as corresponding to the number of particles N within space V we write

$$N = 0.96489 \frac{V}{\log V}. \qquad \text{III}$$

Here it must be observed that V does not denote the actual dimensional volume of the universe, but only its arithmetical value in cm^3. V as well as the other constants we shall encounter subsequently are arithmetical numbers without dimensions, pertaining to the ideal scheme that precedes the actual creation of the material universe; that scheme being arbitrarily but consistently represented by the CGS-system of physical science. The volume of space $V = R^3\pi^2$. The value of R we have to derive from the velocity of recession v. For v we adopt the value 792.6 km/sec/mp, which gives $R = 6.77 \cdot 10^{26}$ cm and $V = 3.06195 \cdot 10^{81}$ cm^3.

Introducing these values into equation III we find

$$N = 1.5746 \cdot 10^{79}, \qquad \text{IV}$$

the value of Eddington for the Cosmical Number.[6] Considering the titanic figures involved, the odds against the coincidence being a matter of pure chance are overwhelming. What Jeans says[7] with reference to the planetary orbits (where only 2000 are involved and the odds are $2^{1999} - 1$ to one against their arrangement being a matter of pure chance), is billionfold true in our connection. The odds that the prime number distribution of matter is *not* a matter of pure chance, are far greater than those in favor of well attested historical events: it is more certain that some definite cause underlies that distribution of particles than it is, for instance, that the Athenians won the battle of Marathon or that Queen Ann is dead. *x is to P_x as V is to N. The distribution of particles equals that of primes.*

The value of the highly important factor[8] log V, which links the volume of the universe to the number of particles is 13.7^2. Our Theorem then reads

$$N = 0.96489 \frac{V}{13.7^2}, \qquad \text{V}$$

[6] The exact value of N depends on that of v. The exact value of v is 792.614, the exact value of R $6.76965 \cdot 10^{26}$.

[7] *Astronomy and Cosmogony*, Cambridge: At the University Press, 1929, page 4.

[8] The exact value of log V depends, like that of N, of the value V, or v. For v = 792.614 log V = 13.69775^2. The factor a is of no importance in this connection. If it were taken as 1.00000 log V would be 13.69629^2.

or

$$N = 96.489 \frac{V}{137^2}. \qquad \text{VI}$$

The number 13.7^2 can be resolved into the numerical factors

$$13.7^2 = \frac{1846.61 \cdot 980.665 \cdot 2.99796 \cdot 10^{10}}{6.06436 \cdot 10^{23} \cdot 4.77 \cdot 10^{-10}}. \qquad \text{VII}$$

All these numerical factors represent the arithmetical values of physical constants only, and indeed of as many fundamental ones as could be wished for. Every constant not present can be deduced from the ones present. 1846.61 is the mass-ratio, 980.665 the arithmetical value of the acceleration due to gravity, $2.99796 \cdot 10^{10}$ that of the velocity of light; $6.06436 \cdot 10^{23}$ that of Avogadro's (Lochschmidt's) number, $4.77 \cdot 10^{-10}$ that of the electronic charge. As is seen, *the number 13.7^2 contains the arithmetical values of all the constants of nature.*

Likewise, the number 9648.9 can be resolved into the following numerical components

$$9648.9 = \frac{6.06436 \cdot 10^{23} \cdot 4.77 \cdot 10^{-10}}{2.99796 \cdot 10^{10}}. \qquad \text{VIII}$$

As is seen, a in the Prime Number Theorem was chosen to be $F \cdot 10^{-4}$, where F denotes Faradays' constant. The similarity of $F \cdot 10^{-4}$ with the average value of a and its correlation and intrinsic complementarity to the number 13.7^2 within the Prime Number Theorem is indeed a remarkable coincidence.

From VII and VIII follows that

$$a = 0.96489 = \frac{1846.61 \cdot 980.665}{13.7^2 \cdot 10^4}. \qquad \text{IX}$$

Combining equation V and IX we get the result

$$\frac{137^4}{1846.61} = \frac{980.665\, V}{N}. \qquad \text{X}$$

Equation IX combines all the three pure number constants of physics in a simple formula. The appearance of the value 980.665 as only additional factor to the ones treated from the beginning of this study seems to be an interesting reminder of the fact that all our values are based on the CGS-system. It is the representative of our system of reference; the earth.

The equality between the distribution of primes and of particles solves several problems of the distribution of matter in space, especially those of the initial distribution and of the spiral arms of nebulae. The first distribution was either even or uneven. If it was even, there is the problem how the first excitement of matter arose that started the world process. If it was uneven the question remains what was the law of that uneven distribution. The prime number distribution of matter solves the problem. It is an intermediary between random distribution and order.

A clue to the solution of the problem of the spiral nebulae can be found when the coördinates of the sky are applied to the prime numbers. The fundamental, so far unsolved, problem of arithmetic is to find a method of determining any prime number as such. To determine the position of a number, that position must be given within the decimal series of all numbers. Since we are only concerned with prime numbers it would be useless to include in our coördinates numbers which obviously are not primes. This is the case with all even numbers, easily recognizable by their last digit or the total of their digits, and all numbers divisible by three, equally easily recognizable by the total of their

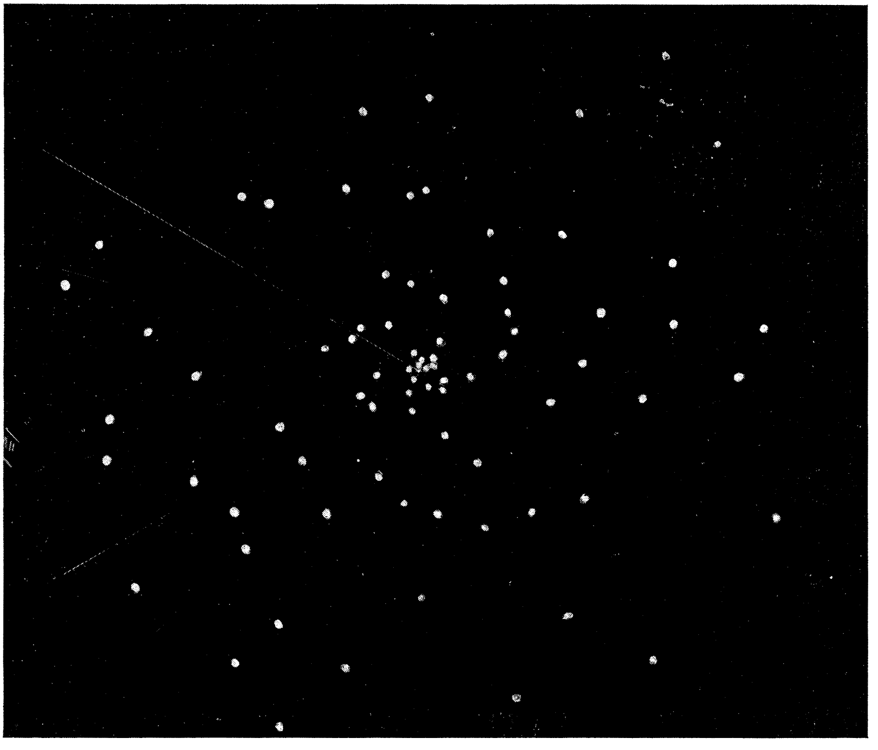

CHART I

digits. Prime numbers can only have one of the six totals of digits 1, 2, 4, 5, 7, 8, and only end with one of the four digits 1, 3, 7, or 9. Arranging the primes into six great groups according to their respective totals of digits and subdividing those six principal sectors again into the four possible sectors corresponding to the respective last digits, we obtain a natural arrangement of primes within 24 sectors. This will be our longitudinal coördinates. Since every prime, in addition, must have a fixed position within the decimal series of numbers, our latitudinal coördinates will be parallels representing the integers. Thus the coördinates of primes correspond by their intrinsic nature to those of the celestial sphere.

It is now easy to insert the primes at their respective places in the scheme of coördinates. The result is the Chart of Primes, seen here. (Chart I.) Though it is extremely incomplete, only showing the first 79 primes, up to 400, it gives a good idea of the resemblance with a stellar chart. It shows the decreasing frequency and the apparent lawless distribution. Further scrutiny however, reveals a striking fact. It is seen that there exist spiral-formed arms of primes, extending in convolutions from and around the center of the chart. This is an

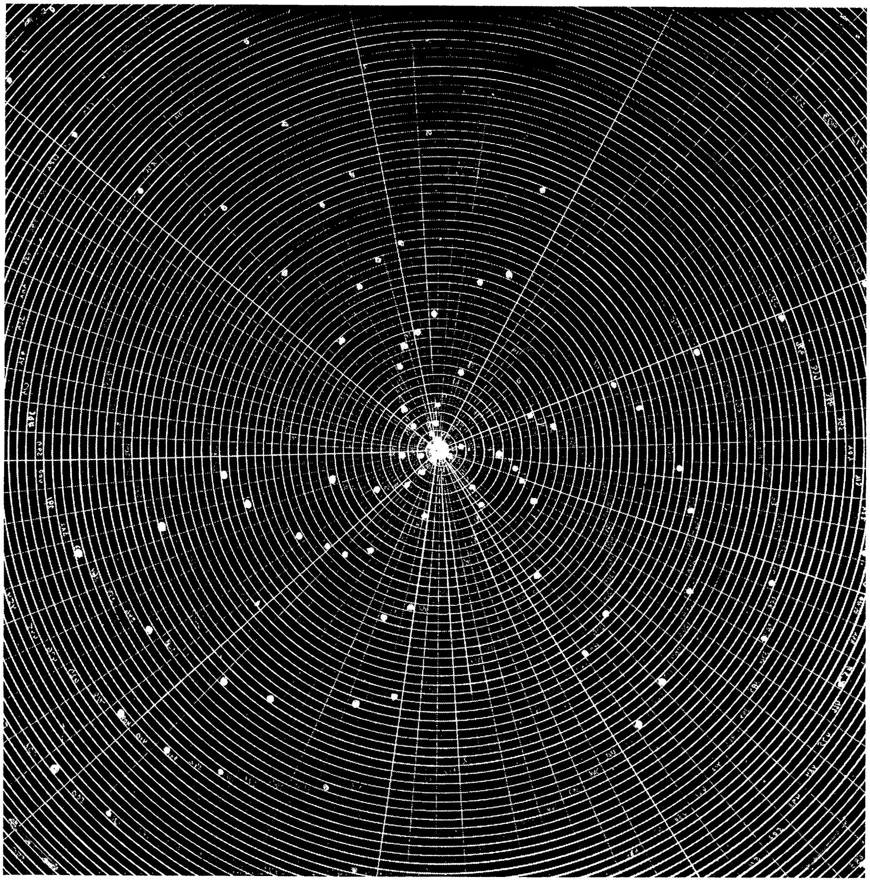

CHART II

exact analogy to the mysterious spiral arms of galactic nebulae, pictured schematically and photographed e.g. in Jeans' *Astronomy and Cosmogony*.[9]

The great problem of the spiral nebulae is, firstly, why the arms have just that particular shape, and secondly, why the length of the arms remains permanently equal to about two convolutions and no more. If the arms were orbits described by ejected matter which has congealed into stars they ought, first of all, be nearly circular, but not open orbits, such as equiangular spirals.

[9] Page 358 ff. There also following quotations.

Secondly, they ought to increase with the length of time, a nebula of average age would have to show many thousand convolutions. The matter seems impossible to explain in terms of known forces, and Jeans is "led to the disconcerting, but almost inevitable conjecture, that the motions in the spiral nebulae must be governed by forces unknown to us." The central points of the nebulae seem to "appear as *points at which matter is being continually created*."[10]

According to our theory the spiral arms present no problem, but follow as a matter of course from the prime number distribution of matter within the celestial coördinates. From the central point the series of numbers, or matter, emerges. The form of arrangement is that of an equiangular spiral, with open orbits. However, that form is only the ideal shape which in the actual nebulae is never attained; as Jeans relates with reference to the measurements of van Pahlen, Groot and Reynolds. Equally, the equiangular spiral shown by the prime number chart is only an approximation to the strictly spiral shape of the form $r = Ae^{\alpha\theta}$. We are, actually, in a position to demonstrate the form of deviation from the ideal spiral. The ideal shape of the equiangular spiral is appearing, when *all* numbers are arranged in the way we have arranged the prime numbers on the chart. Then we obtain no counterpart to any material configuration, but the ideal origin of those configurations. Indeed, Chart II seems to be a perfect representation of the ideal nebula. That ideal scheme of coördinates comprises 9 sectors, corresponding to the 9 possible totals of digits of all numbers, and 10 sub-sectors, according to the 10 possible last digits; i.e., 90 sectors altogether. As is seen in the picture, this arrangement of numbers gives the perfect equiangular spiral, the maternal scheme of matter and primes. Resolving this scheme into its prime factor elements, the three sectors of the numbers divisible by 3 and the 6 sub-sectors not compatible with prime factors disappear. Then it becomes clear why there can only be about two convolutions in the spiral arms of the nebulae and of the prime chart. The original regular convolutions of the ideal chart have been disturbed, the orbits widened; the scarcity of primes increases in such a way as never to allow the convolutions to complete further rounds, but stretching them farther and farther afield. The forces arranging the nebulae in that way are the same forces which otherwise arrange matter in conformity with mathematics. Not, as Jeans says, the forces were unknown to us, only the basic mathematical pattern according to which they have arranged themselves.

New York, N. Y.

[10] Italics mine.