MAT 331, Spring 2010, Problems

1. Use `Maple` to write $x^5 - 6x^3 - 5x - 3x^4 + 18x^2 + 15$ as a product of *exact* linear factors. By exact, I mean you should leave any non-rational factors expressed as radicals; do not approximate terms like $\sqrt{3}$ as 1.73205, etc.

2. Consider the planar curve γ defined by $x^2y^3 + y^2 + y - 2e^x = 0$. Using **only** `Maple`, find the slope of the tangent line to the curve at $(0, 1)$. Then plot the curve and the tangent line on the same graph.
Hint: you might want to use `implicitplot` *from the library* `plots`*. You might find* `implicitdiff` *helpful, too.*

3. Plot the function $f(x) = 2\sin x - x^3 - 5$, for $x \in [-4, 4]$. Find all the zeros of the function with an accuracy of 20 decimal digits. *Hint: See* `Digits`*,* `fsolve`*, and/or* `evalf`*.* Make sure you find *all the zeroes. Justify your answer.*

4. Define a `Maple` function g that, given a positive integer k yields the sum of the first k primes. What is k such that $g(k) \leq 100,000$ but $g(k + 1) > 100,000$? You might find `sum` and `ithprime` helpful.

5. The Fibonacci sequence is a sequence of positive integers defined by recurrence as follows: $F_0 = 1, F_1 = 1$ and for each integer i larger than one, $F_i = F_{i-1} + F_{i-2}$. Use Maple to find the first 100 terms of the sequence without computing these terms one by one (Hint: See Fibonacci). Use the sequence you found to compute a new sequence F_i/F_{i-1} and try to guess what is the limit of that new sequence. EXTRA CREDIT: Prove the sequence F_i/F_{i-1} converges to the limit you guessed (You may need to refine your guess to do so).

6. Find a polynomial of degree 3 passing through the points

$$(-3.3, 1), (-2, 2), (0, 3), (1.1, 4)$$

by two methods: First solve the equations (as in the book). Second, use the build-in `Maple` command. Check that the two solutions agree.

7. (a) Find two polynomials of degree 5 passing through the points

$$(-3.3, 1), (-2, 2), (0, 3), (1.1, 4).$$

   (b) Plot the two polynomials and the points in the same graph.

   (c) Is it possible to find more polynomials of degree 5 passing through the 4 points? Why?

   (d) How many polynomials of degree 3 passing through the 4 points can you find?

8. Find a polynomial which passes through the points

$$(-3, (-9)/4), (-2, (-3)/2), (-1, (-3)/4), (0, 0),$$

$$(1, 3/4), (2, 3/2), (3, 9/4), (4, 3), (5, 15/4), (6, 9/2), (7, 5.35), (8, 6).$$

Then find a piecewise linear function passing through them.

9. Fit the points given in the previous problem to a line using the least square method. Plot the three results (the to of the previous problem, and the one you just obtained) and the points in the same graph. Compare the results. Which one do you think is best and why?

10. Fit the points $(-1.9, -4.7), (-0.8, 1.2), (0.1, 2.8), (1.4, -1.2), (1.8, -3.5)$ by means of a quadratic function $f(x) = ax^2 + bx + c$, using the least square method. First, do this step by step, as we did in class; then, use the built-in `Maple` command, described in the notes. Check that the two solutions agree.

11. Fit the set of points

$$(1.02, -4.30), (1.00, -2.12), (0.99, 0.52), (1.03, 2.51), (1.00, 3.34), (1.02, 5.30)$$

with a line, using the least square method we used in class. You will see that this is not a good fit. Think of a better way to do the fit and use `Maple` to do it. Explain in your solution why you think your better way is better.

12. Fit the points $[[1, 1], [2, 3], [3, 1], [1, -1.1]]$ to a circle.

13. Following the on-line notes, find a set of 25 points which are "approximately" in a cubic and:
    (a) Find the cubic that best fit the points by the least squares method using the CurveFitting package.
    (b) Find the cubic that best fit the points by the least squares method without using the CurveFitting package.
    (c) (EXTRA CREDIT) Find the corresponding spline.

14. *[In this problem use* ***`Maple`*** *only as a word processor. If you're more confortable with paper, you can turn in a paper instead of a* ***`Maple`*** *worksheet.]* Let n points of the form (r_i, r_i^2), $i = 1, 2, \ldots, n$, be given. What is the quadratic function $f(x) = ax^2 + bx + c$ that best fits them? **Prove** your answer. Does it depend on the optimization method (least square or others)?