Homework 1

Section 1.1:

12.

Sol:

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \end{pmatrix} \begin{matrix} \\ \mathrm{II}-2\mathrm{I} \end{matrix} \to \begin{pmatrix} 1 & -2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

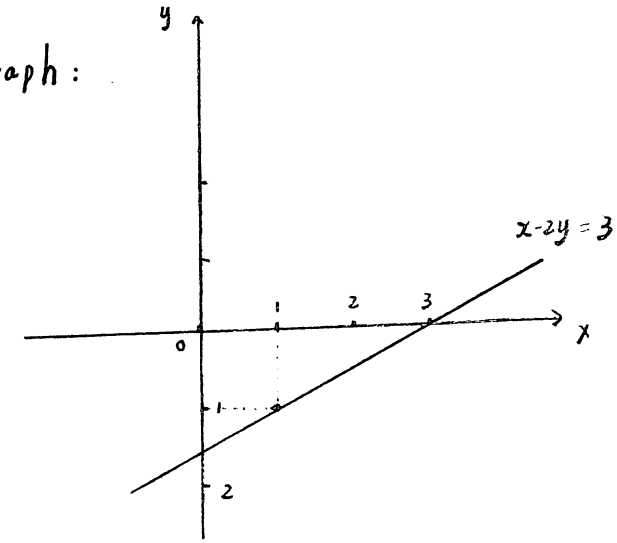
$\Rightarrow x-2y=3$

Set: $y=t \Rightarrow x=3+2t$

Then solutions:

$$\{(3+2t, t) : t \in \mathbb{R}\}$$

Graph:



16.

Sol:

$$\begin{pmatrix} 1 & 4 & 1 \\ 4 & 13 & 7 \\ 7 & 22 & 13 \end{pmatrix} \begin{matrix} \\ \mathrm{II}-4\mathrm{I} \\ \mathrm{III}-4\mathrm{I} \end{matrix} \to \begin{pmatrix} 1 & 4 & 1 \\ 0 & -3 & 3 \\ 0 & -6 & 6 \end{pmatrix} \begin{matrix} \\ \mathrm{II}\div(-3) \\ \mathrm{II}\div(-6) \end{matrix} \to \begin{pmatrix} 1 & 4 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \begin{matrix} \\ \\ \mathrm{III}-\mathrm{II} \end{matrix}$$

$$\to \begin{pmatrix} 1 & 4 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} x+4y+z=0 \\ y-z=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=-(4y+z)=-5z \\ y=z \end{cases}$$

Set $z=t$, $\Rightarrow x=-5t \quad y=t$

Solutions: $\{(-5t, t, t) = t(-5,1,1) : t \in \mathbb{R}\}$

As we can see, it is a line as the intersection of the plane $x+4y+z$ and $y-z=0$ in $\mathbb{R}^3$

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Section 1.2:

16. Sol:

$$\begin{pmatrix} 3 & 6 & 9 & 5 & 25 & 53 \\ 7 & 14 & 21 & 9 & 53 & 105 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{pmatrix} \begin{matrix} \mathrm{I} \\ \mathrm{II} \\ \\ \end{matrix} \to$$

$$\begin{pmatrix} 7 & 14 & 21 & 9 & 53 & 105 \\ 3 & 6 & 9 & 5 & 25 & 53 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{pmatrix} \begin{matrix} \mathrm{I}-2\mathrm{II} \\ \\ \\ \end{matrix} \to$$

$$\begin{pmatrix} 1 & 2 & 3 & -1 & 3 & -1 \\ 3 & 6 & 9 & 5 & 25 & 53 \\ -4 & -8 & -12 & 5 & -10 & 11 \end{pmatrix} \begin{matrix} \\ \mathrm{II}-3\mathrm{I} \\ \mathrm{III}+4\mathrm{I} \end{matrix} \to$$

$$\begin{pmatrix} 1 & 2 & 3 & -1 & 3 & -1 \\ 0 & 0 & 0 & 8 & 16 & 56 \\ 0 & 0 & 0 & 1 & 2 & 7 \end{pmatrix} \begin{matrix} \\ \mathrm{II}\div 8 \\ \\ \end{matrix} \to$$

$$\begin{pmatrix} 1 & 2 & 3 & -1 & 3 & -1 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 1 & 2 & 7 \end{pmatrix} \begin{matrix} \\ \\ \mathrm{III}-\mathrm{II} \end{matrix} \to$$

$$\begin{pmatrix} 1 & 2 & 3 & -1 & 3 & -1 \\ 0 & 0 & 0 & 1 & 2 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} x_1+2x_2+3x_3-x_4+3x_5=-1 \\ x_4+2x_5=7 \end{cases}$$

Set $x_5=r$, $x_3=s$, $x_2=t$

$\Rightarrow x_4=7-2x_5=7-2r$

$$\begin{aligned} x_1 &= -2t-3s+x_4-3x_5-1 \\ &= -2t-3s+7-5r-1 \\ &= 6-2t-3s-5r \end{aligned}$$

Solusions:

$$\{(6-2t-3s-5r, t, s, 7-2r, r) : (r,s,t) \in \mathbb{R}^3\}$$

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