

MAT 132 Calculus II

Sample Midterm 2

Nov 9, 2006

1. (5 points) Let $f(x) = \frac{1}{x+1}$
 - (a) Find the average value of f in $[0, e^3 - 1]$.
 - (b) Find c in $[0, e^3 - 1]$ such that $f(c)$ is equal to the average value of f in $[0, e^3 - 1]$.
2. (6 points) A spring has a natural length of 5cm. Suppose 2 joules of work is needed to stretch a spring from its natural length to a length of 30 cm.

Find the length the spring will be stretched starting from a length of 15cm if the work needed in this case is 10 joules.

3. (6 points) Verify that $y = x \tan^{-1}(x)$ satisfies the differential equation

$$x(1 + x^2)y'' - 2y' + \frac{2}{x}y = 0$$

4. (6 points) Solve the initial value problem

$$\frac{1}{\cos(x)} \frac{dy}{dx} = xy \text{ with } y(0) = 3.$$

5. (6 points) Cobalt-60 has a half life of 5.24 years.
 - (a) Find the mass that remains from a 100-mg sample after 20 years.
 - (b) How long would it take for the mass to decay to 1 mg?
6. (5 points) A direction field for the differential equation $y' = x^2 - y^2$ is shown in Figure 1. Sketch the solution of the initial-value problem,

$$y' = x^2 - y^2, y(0) = -1$$

7. (5 points) Find the general solution of each of the following differential equations.

- (a) $2y'' - 4y' - 7y = 0$

- (b) $y'' + 12y' + 36y = 0$

8. (6 points) Solve $y'' - 2y' - 3y = 0$ with $y(0) = 3$, $y'(0) = -3$.

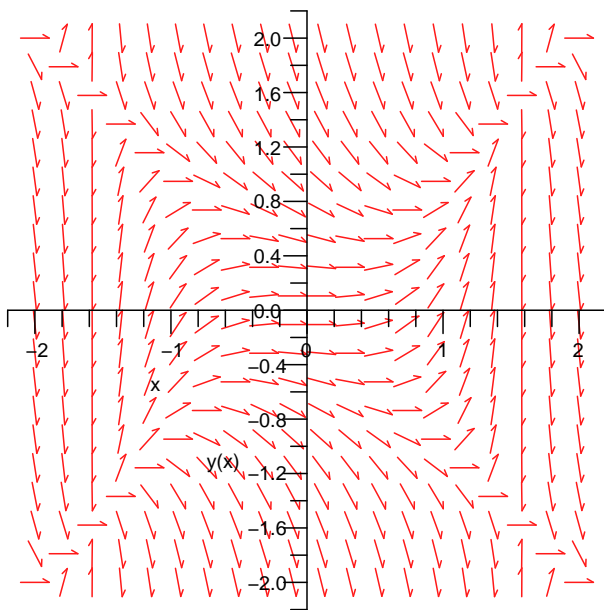


Figure 1: Graph for problem 6