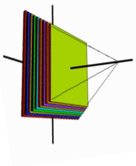
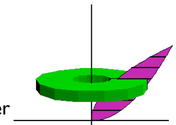
MAT 132

More review and the cylindrical shells method

- Section (a **(square in this case)**

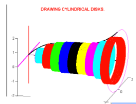


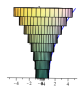
- Washer



(a flat ring)

- Disks (solid of revolution) **a circle**





Volume of a slice

$A(x).\Delta(x)$

Volume of the solid

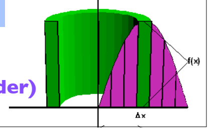
$\int_a^b A(x)\,dx$

A(x) is perpendicular to the direction of x.

Here, half a shell is shown.

- Shell

(a cylinder)



One of the solutions of $y=3x-x^3$ (x as a function of y.)

$$\frac{1}{2}\left(-4y+4\sqrt{-4+y^2}\right)^{1/3}+\frac{2}{\left(-4y+4\sqrt{-4+y^2}\right)^{1/3}}$$

1–12 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

11. $y = 1 + \sec x$, $y = 3$; about $y = 1$

12. $y = x$, $y = \sqrt{x}$; about $x = 2$

(for practice, find also the volume the solid obtained by rotating about the x-axis in 11. and about the y-axis in 12.)

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(2) For each of the following improper integrals:

(i) determine whether or not it converges.

(ii) Evaluate those that converge.

(a) $\int_0^4 x(16-x^2)^{-3/2}\mathrm{dx}$.

(b) $\int_1^\infty \frac{\ln(x)}{x}\mathrm{dx}$. (CORRECTED)

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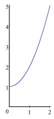
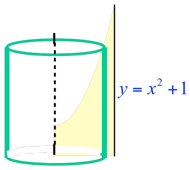
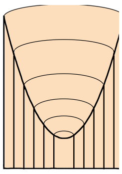
1. Consider the region R bounded by the x axis and the graph of $y = 4x - x^2$ about the y axis. Find the volume of the solid obtained by rotating R about the y-axis

Note:
We rotate about the y-axis
The variable in the integrand is x.

Demo







The volume of a thin, hollow cylinder is (lateral surface area of cylinder).(thickness)= (2. π .x. heigh(x) . thickness)= 2.π.x.f(x).dx

Find the volume of the solid obtained by revolting about the y-axis, the region bounded by $y=x^2+1$, the y axis and x=2

Shell Method (compute the volume of a solid obtained by revolting a region R in the first quadrant about the y-axis)

- Draw the region R.
- Sketch a line segment (in R) parallel to the y-axis.
- Label: segment length (shell heigh) and distance from the y-axis (shell radius).
- Determine the limits of integration.
- Integrate 2 π (shell radius)(shell heigh)=π.x.f(x) over the limits of integration you found.

Find the volume of the solid generated by revolving about the y-axis the region bounded by the curve $y=\sin(x^2)$, the x-axis and the lines $x=\sqrt{\pi}/2$ and $x=\sqrt{\pi}$.

- Find the volume of the solid obtained by rotating about the y-axis the region in the first quadrant bounded by the parabolas $y=2-x^2$ and $y=x^2$.

- Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y=2x^2-x^3$ and y=0.

The region bounded by the curve $y=x^2$, the x-axis and the line x=2 is revolved about the y-axis. Find the volume of the solid generated in two ways: using the shell method and the washer method.

Answer= 8π