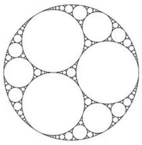
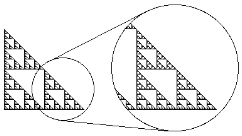
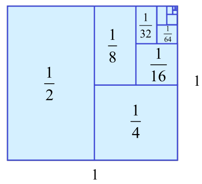




MAT 132 8.2- Series

Consider the sequence defined explicitely by the formula $a_n=1/2^n$.
Define a new sequence:
$s_1=a_1$,
$s_2=a_1+a_2$,
$s_2=a_1+a_2+a_3$,
...
$s_n=a_1+a_2+..+a_n$,
Is $\{s_n\}$ convergent?



Consider a sequence $\{a_n\}$. We associate with $\{a_n\}$ a new sequence $\{s_n\}$, with general term
$s_n=a_1+a_2+..+a_n$,
The sequence $\{s_n\}$ is called an *infinite series*.
If the sequence $\{s_n\}$ is convergent to a number s then we write

$$\sum_{n=1}^{\infty} a_n = s$$

The number s is called the *sum* of the series.

Example: give the first three terms of the series associated to the sequences below , determine whether they are convergent and if so, find the sum.

a. $a_n=(-1)^n$.

b. $a_n=n/(n+1)$

c. $a_n=n$

d. $a_n=1/(n.(n+1))$.

e. $a_n=1/3^n$.

Hint: Recall

$$(a^n-b^n)=(a-b)(a^{n-1}+a^{n-2}b+\cdot+ab^{n-2}+b^{n-1})$$

$\sum_{n=1}^{\infty} ar^{n-1}$ Determine whether the each of the series below is a geometric series. If is, find a formula for the general term and a formula for the sum.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots \qquad \sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{3}{5}-\frac{12}{25}+\frac{48}{125}-\frac{192}{625}+\ldots \qquad \sum_{n=1}^{\infty}\frac{3}{5}\left(-\frac{4}{5}\right)^{n-1}$$

$$\frac{2}{3}+\frac{4}{3}+\frac{8}{3}+\frac{16}{3}+\ldots \qquad \sum_{n=1}^{\infty}\frac{2}{3}(2)^{n-1}$$

The sum of a geometric series

$$s_n = a + ar + ar^2 + ar^3 + \ldots ar^{n-1}$$ Sum of n terms

$$rs_n = ar + ar^2 + ar^3 + \ldots ar^n$$ Multiply each term by r

$$s_n - rs_n = a - ar^n$$ subtract

$$s_n = \frac{a-ar^n}{1-r} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

$$if\ |r|<1, \quad r^n \rightarrow 0\ as\ n \rightarrow \infty.$$

Geometric series converges to $s_n = \frac{a}{1-r}, |r|<1$

If r>1 the geometric series diverges.

Find, if possible, the sum of the geometric series below.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots \qquad \sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{3}{5}-\frac{12}{25}+\frac{48}{125}-\frac{192}{625}+\ldots \qquad \sum_{n=1}^{\infty}\frac{3}{5}\left(-\frac{4}{5}\right)^{n-1}$$

$$\frac{2}{3}+\frac{4}{3}+\frac{8}{3}+\frac{16}{3}+\ldots \qquad \sum_{n=1}^{\infty}\frac{2}{3}(2)^{n-1}$$

Recall: An geometric sequence has the form $a_n = a_1 \cdot r^{n-1}$

- Write the number $3.1\overline{234}$ as a ratio of integers.

Important example.

- The series associated with the sequence $a_n=1/n$ is called the harmonic series.
- Determine whether the harmonic series is convergent.

Theorem

- If a series associated to the sequence $\{a_n\}$ is convergent, then $a_n \to 0$ when $n \to \infty$.

Question: if $a_n=1/n$ then $a_n \to 0$ when $n \to \infty$. Does this imply that the harmonic series is convergence.

What does this theorem tell us?
If a_n does not converge to 0 when $n \to \infty$ then the series diverges.
If $a_n \to 0$ when $n \to \infty$ the series may converge (but we need more work to prove it or disprove it)

8 Theorem If $\Sigma\, a_n$ and $\Sigma\, b_n$ are convergent series, then so are the series $\Sigma\, ca_n$ (where c is a constant), $\Sigma\,(a_n + b_n)$, and $\Sigma\,(a_n - b_n)$, and

(i) $\sum_{n=1}^{\infty} ca_n = c\sum_{n=1}^{\infty} a_n$ (ii) $\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$

(iii) $\sum_{n=1}^{\infty}(a_n - b_n) = \sum_{n=1}^{\infty} a_n - \sum_{n=1}^{\infty} b_n$

Find the area and the perimeter of the Koch Snowflake